Interbank Hedging and Systemic Risk: The Role of Renegotiation Breakdowns *

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Abstract

Banks use over-the-counter derivative (OTCD) contracts for sharing the risks of their asset streams. OTCDs are composed of an optimal combination of interbank loans, asset swaps and credit default swaps (CDS). The settlement of ex-post realized claims are renegotiated by participating banks after defaults by one or more banks to minimize the systemic impact of the default. The bailout by a solvent bank of an insolvent one creates a positive externality for all other solvent banks in the system with claims due from the insolvent bank. The lack of coordination among the solvent banks leads to an inefficient liquidation policy (system-wide runs) even though we allow mergers among them to potentially internalize the positive externality. Interbanks loans and CDS contracts have relatively lower incentive costs than swaps, but the former has the largest distress costs without renegotiations. With renegotiations, the payoffs of interbank loans and CDS are similar in strong bankruptcy regimes, while the latter dominate in weak regimes. Asset swaps provide the best hedging and are optimal for banks without incentive problems. Hedging with optimal OTCD contracts creates a tradeoff between the amount of credit risk and systemic risk (liquidation spillover) in the banking system.

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Introduction

The exponential growth of the market for over-the-counter derivative (OTCD) products in the past decade that has facilitated the transfer of risk between financial institutions has been one of the more remarkable trends in finance. The top panel of Figure 1 shows the notional amounts of total OTCD and asset swap contracts. The bottom panel shows the notional amounts of credit default swaps (CDS). CDS contracts have been a relatively recent innovation, but their share of the OTCD market has increased rapidly. The recent increase in systemic concerns emanating from the defaults of subprime mortgage loans and resulting in perhaps the most severe banking crisis since the Great Depression. has left both academics and practitioners searching for rational economic explanations for both its severity and its rapid transmission. Many have explicitly put the blame on the huge mass of outstanding derivative contracts and in particular CDS contracts that has substantially changed the structure of financial markets. Indeed the role of derivative products in increasing systemic risk has been hotly debated over the past decade.[1] [2]

While it is a reasonable conjecture that the increase in financial linkages from the OTCs would increase the transmission channels by which shocks to one financial institution or sector can adversely affect other financial institutions thus generating systemic risk, taking stock of recent developments we highlight the elements of the current financial crisis that we build into an economic model to enhance an understanding of the role of OTC derviatives in increasing the severity of the crises and its rapid transmission.

[1] While the exact definition of systemic risk remains up to debate [see Schwarcz (2007) for alternative definitions], one popular definition that has emerged is the risk that a default by one financial institution will have repercussions on other institutions due to the interlocking nature of financial markets. For example, a default by bank A on financial contracts on which it is due payments to bank B will affect affect the ability of B to come good on its obligations to bank C, and so on, in a cascading domino effect.

[2] In this term as the Federal Reserve Chairman, Alan Greenspan argued against regulation of OTCD markets as evidenced in these following quotes:

> What we have found over the years in the marketplace is that derivatives have been an extraordinarily useful vehicle to transfer risk from those who shouldn'tt be taking it to those who are willing to and are capable of doing so. We think it would be a mistake to more deeply regulate the contracts, [Alan Greenspan to the Senate Banking Committee in 2003].

He added that the growth of the derviatives market did not pose dangers to the financial system.

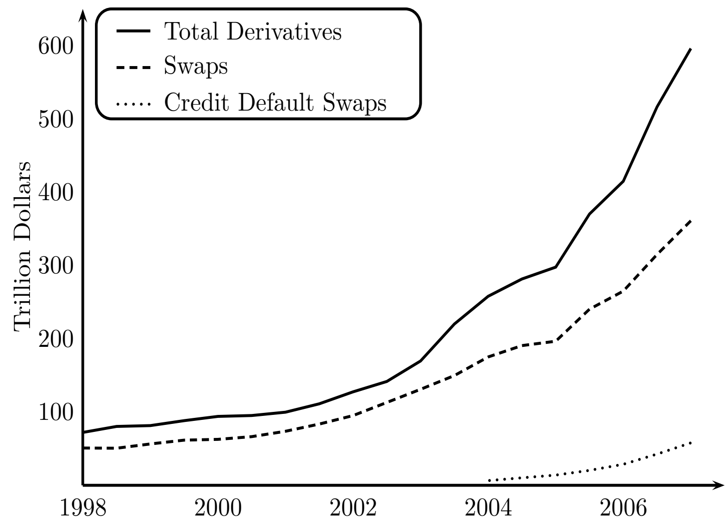
> Not only have individual financial institutions become less vulnerable to shocks from underlying risk factors, but also the financial system as a whole has become more resilient. [Alan Greenspan, 2004]

In contrast Warren Buffet warned in 2003:

> Derivatives are financial weapons of mass destruction, carrying dangers that, while now latent, are potentially lethal. [Warren Buffet, 2003]

In 2008, Greenspan told Congress that he was "shocked" by the breakdown in the U.S. credit markets and he had been "partially wrong" to resist regulation of derivatives markets.

Figure 1: Ratio of Outstanding OTC Derivatives to Asset Values at Banks



The figure shows the notional amounts outstanding of over-the-counter derivatives (OTCDs) held at banks and dealers in the G10 countries Source: BIS Quarterly Review 2008.

1. OTC derivatives contracts are held by a small number of banks. In contrast to exchange traded derivatives, most such contracts are not actively traded and hence their prices are determined by negotiation among its counterparties. In addition, since OTC derivatives are not marked-to-market they expose the banks to the risk that their counterparties are unable to fulfill their obligations.

2. Hedging with the OTC derivatives partially separates the origination and eventual effective ownership of the underlying financial assets and thus reduces the incentives for originating institutions to maintain the quality of the assets originated.

3. Financial institutions attempt to renegotiate the payments on outstanding derivative claims if one or more counterparties is insolvent to lower dead weight liquidation costs and increase recoveries. One means of insolvency management has been the explicit partial or complete mergers between banks that can be used to "net" out some of these claims.[3]

[3] Prominent recent examples include the purchase of Bear Sterns by J. P. Morgan, and Merrill Lynch by Bank of America in 2008. Several other large institutions such as Citibank have been actively attempting to sell large parts of their assets to other institutions such as Sovereign Wealth Funds and better capitalized institutions in Asia.

4. Financial institutions in trouble often face liquidation even when they have a net positive value to the other banks in the system. This creates the need for a regulator to often coordinate the actions of the counterparties of the troubled bank.[4]

5. The rating agencies have been blamed for providing overly optimistic ratings for these OTCs, often ignoring the systemic component of risk over and above the inherent credit risks that are present in these contracts.

The goal of building a model with the above listed features is to ask several questions: Are OTC derivatives effective vehicles for risk sharing, and if so, should they be more popular than interbank loans? In particular will they provide protection to distressed firms in their deepest crises? Will they lead to inefficient liquidation decisions of firms in distress? What sorts of OTC derivatives lead to a larger chance of ineffective risk sharing and inefficient liquidation decisions? Does the asset transfer lower the quality of the underlying assets that are created by the financial institutions? Do profit maximizing banks transfer an efficient level of assets and does transfer lead to increased systemic risk? Is there a tradeoff between credit risk and systemic risk of financial institutions? Finally, does the bankruptcy regime in which the financial institutions operate in affect the credit and systemic risk created by OTC derivatives?

Our model has N financial institutions (FIs), who could be traditional banks, investment banks, mortgage or finance companies, mutual funds, or hedge funds. We will simply refer to all such FIs as "banks". Each bank has ownership to a stream of assets, which in the credit risk literature is often referred to as the 'unlevered' asset value. The asset stream reflects the business of the bank in making loans and purchasing investments outside the banking system. It also has liabilities or deposits, which are senior to all other claims.[5] Banks attempt to diversify the risk in their asset stream by engaging in interbank OTC transactions. OTC derivatives are complex financial contracts that are made of simpler contracts such as pure loans, asset swaps, and credit default swaps. These contracts help to smooth out the banks' profits. If at maturity of these contracts, all banks are solvent, then claims are settled as contracted. However, if one or more banks are insolvent, we assume that these banks attempt to renegotiate the contractual terms of the contracts ex post to increase their recovery rate by avoiding liquidation costs. If renegotiations among the banks fail, then we assume that there is a well established bankruptcy code in the economy which determines

[4] In the case of LTCM in 1998, the Federal Reserve Bank of New York stepped in and coordinated a bailout of the hedge fund by its counterparties.

[5] To highlight the novelty of the economic mechanism introduced in this paper, we assume a two-period model, so that there are no short term liquidity problems that lead to bank runs as has been illustrated in a large literature starting with Diamond and Dybvig (1983). Allen and Gale (2000) analyze systemic risk in such a setting.

how assets of the system of banks are divided among them when they have interbank OTC claims with each other. We describe this bankruptcy procedure next.

For standard debt claims, most papers assume that there is a bankruptcy code that maintains absolute priority and limited liability for the equity holders of the banks. Unlike the code for standard debt claims, the code for interbank claims must *simultaneously* solve for the a set of 'clearing' payments of each bank that satisfy a fixed point condition: receiving pro rata shares of these claims from all other banks, each bank is able to make the payment required of it. Note that imposing this procedure on the banks leads to systemic risk, as banks that are solvent but receive less than full payments on their interbank OTC claims from other banks are unable to make full payments on their commitments, and hence "pass on" their troubles to banks they must make payments to. The clearing system we use generalizes the work of Eisenberg and Noe (2001) by introducing liquidation costs.[6]

The process of renegotiation is modeled as an N player noncooperative bargaining game that endogenously determines the recoveries on OTC contracts for all banks. Potential recovery on defaulting OTC contracts is higher and systemic risk is lower when banks that have positive equity but are unable to make all OTC payments, are *not liquidated* as the bankruptcy process defined above requires. However, if renegotiations among the banks fail, then the regulator steps in and imposes the bankruptcy code on the banks. Banks each attempt to maximize their personal recoveries by threatening to force other banks into the bankruptcy process. A significant contribution of our paper is to show that an efficient liquidation policy is not always possible as renegotiations are not always successful. The model reveals some interesting circumstances that lead to renegotiation breakdowns: In periods when an insolvent bank has OTC payments due to more than one solvent bank, each of the solvent banks is able to credibly threaten to 'run' with its due payment from the remaining banks because the latter find it in their interest to jointly (by merging assets and liabilities) let the first mover run rather then to force the liquidation of the insolvent bank and obtain low recovery on their claims. There are cases when an insolvent firm can provide net equity to the collection of all solvent banks in the system, but no individual bank will bail it out since all the benefits of the bailout accrue to banks who do not join in the bailout. In this sense, the systemic runs from banks is a coordination failure among the solvent banks in the system, and is similar to the coordination failure among depositors in the bank model of Diamond and Dybvig (1983). However, our analysis is richer than the bank runs literature because we allow for possible mergers

[6] The framework has been used in Elsinger, Lehar, and Summer (2006) to determine the amount of systemic risk in a set of Austrian banks with interbank loans and a rich set of assets. Shin (2006) uses the framework to study the general equilibrium effects of increases and decreases in leverage.

between the solvent banks to overcome the coordination failure. The lack of incentive compatibility of the efficient merging policy however leaves the coordination problem unsolved. [7] Our analysis shows that large systemic episodes are far more likely to happen due to renegotiation breakdowns than simply the periods in which banks all receive negative shocks.

A crucial feature of our model is that banks must exert effort to maintain the quality of their asset streams, and this creates an interesting interaction between the provision of incentives and the risk management of system-wide runs. When banks hedge, they transfer the rights to their asset streams to other banks, and thus lose the incentive to maintain asset quality. In equilibirum this implies that banks do not fully hedge and there is residual risk in the system. Of the three types of OTCD contracts considered, the incentive effect is the most severe with asset swaps, and the least with interbank loans. The latter however have the most inflexible payment schedules and are the worst for risk management in the absence of renegotiations. With renegotiations, interbank loans and CDS contracts have very similar payouts in liquidation events in strong bankruptcy regimes.[8] In strong bankruptcy regimes the cost of interbank loans in low and these contracts used optimally can be used to block all inefficient liqudiations (system runs). In weak regimes howver, their cost is high and banks use CDS contracts instead. The latter do not fully block all runs. Asset swaps are used more by banks with fewer incentive problems. Comparing equilibria in the economy with and without OTC derivatives we find that banks profits and social welfare are higher in the presence of OTC derivatives, the effect on credit risk among the banks is lower, and yet the systemic risk is invariably higher as financial distress spreads in states with renegotiation breakdown.

The paper also sheds light on the underestimation of credit risk by standard structural form models and the comovement of credit risk across firms. Huang and Huang (2003) show that once standard credit risk models are calibrated to match historical leverage ratios, volatilities, and default probabilities of alternative rating categories, they predict credit spreads much smaller than those observed historically. On a similar note Collin-Dufresne, Goldstein, and Martin (2001) show that credit spreads of different firms, both financial and nonfinancial, seem to have excess comovement after accounting for the leverage ratios of firms, and their asset volatilities (together comprising the distance-to-default) of firms. Therefore, measures of credit risk seem to be more correlated than the inputs of the structural form model of credit risk. Our model provides an explanation of

[7] Major extensions of this bank run framework to study systemic risk due to liquidity shocks with general networks are in Allen and Gale (2000) (for a survey see Allen and Babus (2008)). As in our paper, Brusco and Castiglionesi (2007) introduce moral hazard issues to this framework. These papers also do not consider merging as a strategy for ex-post distress resolution as a means of solving the coordination problem.

[8] Weak and strong bankruptcy regimes are identified as alternative fixed points of the clearing vector described above that have the minimum and maximum payments by all banks.

both findings for financial institutions. Calibrating the models to a firm's capital structure without accounting for the systemic risk from off balance sheet OTC linkages will underestimate the credit risk of the bank and its correlation with troubles at other banks. Default probabilities and correlation of bank liquidations in our model is very close to the correlation of their asset streams if there is no interbank hedging but are significantly higher once we model systemic risk with renegotiation breakdowns. The large systemic concern that arises from credit risk transfer in our model also sheds light on the underestimation of credit risk of structured investment products by the rating agencies.

Related Literatures

The research closest to ours is the work on OTC markets in Duffie, Garleanu, and Pederson (2008) and Duffie, Garleanu, and Pederson (2005), which include search and bargaining as important elements of valuation in these markets. These papers however only study bilateral bargaining and therefore do not have the breakdowns modeled in this paper. Their work also does not address the systemic implications of these securities.

While market participants and policy makers have been concerned about systemic risk, the limited empirical work on the topic has found it to be a very low probability risk. Earlier simulation studies analyzing interbank exposures such as Humphrey (1986), Angelini, Maresca, and Russo (1996), Sheldon and Maurer (1998), Furfine (2003), Degryse and Nguyen (2004), Wells (2002), and Upper and Worms (2004) investigate contagious defaults that result from the hypothetical failure of a single institution. These papers take a given set of interbank exposures, assume some financial institutions to default and then mechanically clear the interbank market and record which banks are dragged into insolvency. Such analyses are able to capture the effect of idiosyncratic bank failures (e.g., due to fraud).

This approach can be seen as isolating one source of systemic risk, namely, interbank linkages and ignoring the other: correlation in the banks' exposures. Elsinger, Lehar, and Summer (2006) study the credit risk in the Austrian Banking system. They model macroeconomic shocks that hit all banks loan and trading portfolios simultaneously. When defaults occur they analyze how they propagate through the network of interbank exposures and find that the correlation of bank exposures more relevant to generating multiple defaults, than contagion.

Our paper also contributes to the literature on the renegotiation of debt contracts. In most papers a solution to the bargaining game at the time of renegotiation always exists due to the special assumptions made in these papers. Several papers assume that players are able to make "take-it-or-leave-it offers" with exogenous bargaining strengths [see, e.g. Hart and Moore (1998), Garleanu and Zwiebel (2006), and Hackbarth, Hennessy, and Leland (2007)]. Paper such as Bolton and Scharfstein (1996), Rajan and Zingales (1998) and David (2001) endogenize bargaining power using the Shapley value of the game as the solution concept. However, these papers make enough assumptions so that all papers agree to renegotiate with each other and the 'grand coalition' forms. Our work is motivated on recent work on bargaining with externalities by Maskin (2003) who argues that in many real world situations the grand coalition fails to form but a subset of the players agree to divide resources.[9] As in Maskin (2003), we assume the sequential random arrival of banks to a bargaining site where not only the division of the pie but the decisions by banks on who to bargain with is endogenously determined. The random arrival order takes away any first mover advantage to any given bank.

The empirical investigation of renegotiations is still at an early stage but recent work suggests that it is critical to include renegotiations among counterparties to evaluate the effects of financial distress. Roberts and Sufi (2007) provide an empirical analysis of the renegotiation of private credit agreements between US public firms and financial institutions. They report that over 90 percent of long-term debt contracts are renegotiated prior to maturity. However, their method of data collection does not pick up failed renegotiations.

Finally, systemic risk, the risk of insolvencies spreading through the financial system due to interlocking financial contracts, is similar to a related literature on contagion, that also potentially explains why there is a high correlation of financial distress across markets and firms. Pritsker and Kodres (2002) find learning and hedging effects across markets can cause contagion (comovement) in financial markets. Acharya and Yorulmazer (2006) use such an information contagion mechanism to study correlation in bank failure. Collin-Dufresne, Goldstein, and Helwege (2003) use a learning mechanism to explain why credit spreads across all firms increased sharply after the revelations of financial trouble at Enron in 2001. We note, that the channel outlined in this paper to generate correlation in financial distress relies on valuation effects of interbank contracts and holds in a setting with complete information.

The remainder of the paper is structured as follows: In section 1, we provide the structure of the model and the bankruptcy procedure that settles claims in an interbank system. In section 2, we provide a game theoretic analysis of renegotiations among bank, and in section 3 we study optimal OTC contracts. Section 4 concludes. An appendix provides the technical details of an algorithm that solves for values of all banks in a renegotiation for the general case of N banks.

[9] There are several prominent examples of incomplete participation agreements such as the European Union and the Kyoto Protocol.

1 The Model

We consider a simple two period model of a banking system. All contracts are written at date 0 and are settled at date 1.

Assumption 1: There are N identical risk neutral banks, each of which has an 'outside' asset with random value, $\tilde{A}_i$. For simplicity we assume that the N asset distributions are identically distributed

$$A_i \sim \mathrm{LN}(\mu_0 + \mu_1\, h_i - 0.5\,\sigma^2, \sigma),$$

where the assumption of log-normality has no special purpose expect to ensure that the assets always have positive value. Each asset value has a correlation of ρ with each other asset payoff. The term h_i represents the level of effort that each bank can exert to increase the mean of the asset value. We assume that the effort has a cost to each bank of $\gamma\, h_i^2$. The effort is financed by the equity holders and the cost is incurred at date 0. At date 1, this cost is sunk, and hence does not affect settlements.

Assumption 2: Each bank has a senior deposit liability payment due at maturity of L_i. The equity of each bank is $\tilde{e}_i = \tilde{A}_i - L_i$ We assume that all depositors are risk-neutral and have zero time discount. This deposits are senior to all other claims made by the banks. Each bank purchases fairly priced deposit insurance for its deposits. The deposit insurance premium is determined by

$$\omega_i^D = E\left[\mathbf{1}_{\{D_i>0\}}\, \max[L_i - (1-\Phi)\tilde{A}_i - r(\{i\}), 0]\right], \forall i \in \mathcal{N} \tag{1}$$

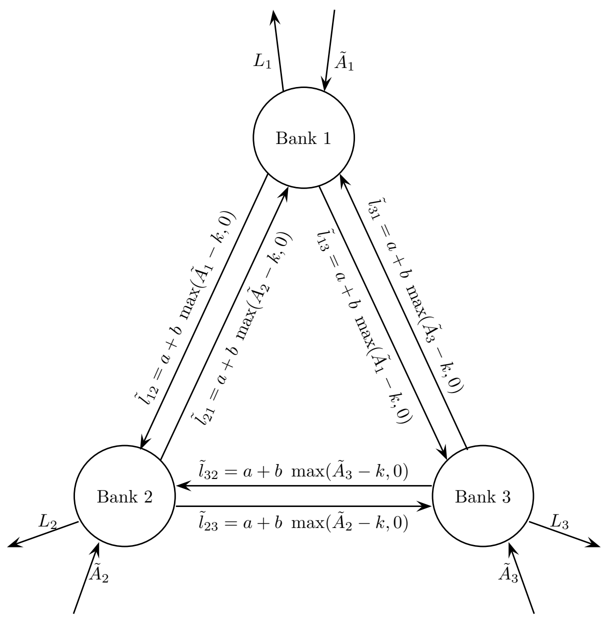
where $\mathbf{1}_{\{D_i>0\}}$ is a liquidation indicator for bank i, which takes the value of 1 whenever the assets of the bank are liquidated, a fraction Φ of the assets are lost upon liquidation because of bankruptcy costs, and $r(\{i\})$ is the payment received by bank i on its interbank claims. The deposit insurance premium is also financed by the equity holders and the cost is incurred at date 0. As the effort costs above, at date 1, this cost is sunk cost as well, and hence does not affect settlements.

Assumption 3: Each bank enters into interbank risk sharing OTC agreements with each other bank, each promising a state contingent payoff of

$$\tilde{l}_{ij} = a + b\tilde{A}_i + c\max[K_j - \tilde{A}_j, 0], \quad \forall i, j \in \mathcal{N}.$$

The interbank claims are junior to the deposits. The OTC contracts are the sum of three parts:

Figure 2: Structure of Banking System with Interbank OTCD Hedges



(i) The component a represents a pure interbank loan, since it offers a fixed payment. The loan is risky since the bank may not be able to repay it in full.

(ii) The component $b\tilde{A}_i$ is the amount of its asset that bank i swaps with bank j in return for the same amount $b\tilde{A}_j$. Notice that the *quid pro quo* exchange arises from the assumption that the banks are ex-ante identical so that the flows have equal discounted values.

(iii) The component $c \max[K_j - \tilde{A}_j, 0]$ represents a reciprocal credit default swap (CDS) arrangement with bank j, where K_j is the face value of a loan made to a firm by bank j and $\tilde{A}_j$ is its market value of this loan. Therefore, bank i agrees to pay bank j the shortfall amount that it faces at date 1. The banks enter into reciprocal CDS agreements so bank i receives $c \max[K_i - \tilde{A}_i, 0]$ from bank j, and the ex-ante premiums cancel by symmetry.

Note that since the ex-ante values of the total payments are identical, entering into such agreements has no impact on the leverage ratios of the banks.

The structure of this network of banks for the case where $N = 3$ is displayed in Figure 2. We will study the optimal ex-post settlement policy of the banks of their deposits as well as their interbank claims.

Assumption 4: The decisions of the banks are made by their equity holders. At the time of settlement of these claims the banks may consider 'mergers' that essentially net out their outside investments and interbank claims Therefore a coalition of banks S has outside assets of $\tilde{A}(S) = \sum_{i \in S} \tilde{A}_i$, liabilities of $L(S) = \sum_{i \in S} L_i$, and interbank claims of $\tilde{l}_{Sj} = \sum_{i \in S} \tilde{l}_{ij}$, and $\tilde{l}_{jS} = \sum_{i \in S} \tilde{l}_{ji}$ for any $j \notin S$.

Assumption 5: The bank faces liquidation costs that are a fraction Φ of the ex-post value of its assets if the equity holders of the bank decide to liquidate the assets.

Assumption 6: At date 1 the N banks attempt to settle all claims. If all banks are solvent ex-post, then all claims are settled in full. Otherwise, the N banks attempt to renegotiate these claims and decide on which banks should be optimally liquidated. If renegotiations break down then we assume that a regulator imposes the bankruptcy code of the economy on these banks, which determines how claims are settled. For the banking system with interbank claims, the division of assets of each bank poses a simultaneous system of conditions, since the amount each bank can pay the other banks depends on how much it receives from these other banks. We call such a system a *clearing* vector, which we describe in detail in section 1.1 below.

1.1 Determination of Clearing Vectors

If at date 1, the banks are unable to settle all claims, then the regulator of the economy steps in and determines a clearing vector of payments that each bank in the system makes in lieu of its promised payments. We generalize the seminal analysis of clearing vectors in Eisenberg and Noe (2001) to include liquidation costs. In addition, we analyze the clearing vectors when banks consider mergers as in Assumption 5 to resolve all financial claims before proceeding to the regulator for a resolution of claims. We denote the complete set of banks with the set $\mathcal{N} = \{1, ..., N\}$. If the banks consider mergers then the resulting set is $\mathcal{F} = \{1, ., f, .., F\}$, where $f \in \mathcal{F}$ consists of one or more merged banks in $\mathcal{N}$, and is $\mathcal{F}$ is thus a "partition" of the original set $\mathcal{N}$. When modeling renegotiations among different banks in the following sections, we will analyze the strategy of various merged banks. Here we will characterize the clearing vector for the general partition $\mathcal{F}$, which for the special case that all banks approach the regulator without merging leads to the clearing vector for the original set $\mathcal{N}$. We therefore generalize the notation of Eisenberg and Noe (2001) to include the superscript $\mathcal{F}$ to denote that the clearing vector is conditional on the banks' merging strategies.

Let $\tilde{d}^{\mathcal{F}}(\{i\}) = \sum_{j=1}^{F} \tilde{l}_{ij}^{\mathcal{F}}$, be the total obligations of the merged bank i in the partition $\mathcal{F}$. We define the relative liabilities matrix of the partition $\mathcal{F}$ as $\tilde{\Pi}^{\mathcal{F}}$ with elements

$$\begin{aligned} \tilde{\Pi}_{ij}^{\mathcal{F}} &= \frac{\tilde{l}_{ij}^{\mathcal{F}}}{\tilde{d}^{\mathcal{F}}(\{i\})} \quad \text{if } \tilde{d}^{\mathcal{F}}(\{i\}) > 0 \\ &= 0 \qquad\qquad \text{otherwise.} \end{aligned}$$

Let $\tilde{p}^{\mathcal{F}}$ be the F vector of payments that each bank makes. Then, the vector of clearing payments received by the banks are given by the vector $\tilde{r}^{\mathcal{F}} = (\tilde{\Pi}^{\mathcal{F}})' \cdot p^{\mathcal{F}}$. Then the clearing vector $p^{\mathcal{F}}$ for this banking system must satisfy

$$\tilde{p}^{\mathcal{F}}(\{i\}) = \min\left[d^{\mathcal{F}}(\{i\}), \max\left(\tilde{A}_i^{\mathcal{F}} - \Phi \tilde{A}_i^{\mathcal{F}} \mathbf{1}_{\tilde{p}^{\mathcal{F}}(\{i\})<d^{\mathcal{F}}(\{i\})} + \tilde{r}^{\mathcal{F}}(\{i\}) - L_i^{\mathcal{F}}, 0\right)\right], \forall i \in \mathcal{F}. \tag{2}$$

The definition states that either bank i makes its full interbank payment of $d^{\mathcal{F}}(\{i\})$, or the regulator will liquidate its assets with a proportional liquidation cost of Φ and these proceeds are used along with the payments that i receives from the other banks to first pay off the deposit holders, and the remaining amount is paid to the other banks in settlement of its interbank claims. This can be written more compactly as

$$\tilde{p}^{\mathcal{F}} = \min\left[\tilde{d}^{\mathcal{F}}, \max\left[\tilde{A}^{\mathcal{F}} - \Phi\, \tilde{A}^{\mathcal{F}} \mathbf{1}_{\tilde{p}^{\mathcal{F}}<d^{\mathcal{F}}} + (\tilde{\Pi}^{\mathcal{F}})' \cdot \tilde{p}^{\mathcal{F}} - L^{\mathcal{F}}, 0\right)\right], \tag{3}$$

where $\max$, $\min$, and $\mathbf{1}$ denote the component wise maximum,minimum, and indicator functions respectively. The right hand side of this equation can be written as a vector valued mapping $\Psi(\tilde{p})$. Stated alternatively, the clearing vector is the fixed point of this mapping. It is straightforward to show by Tarski's fixed point theorem that there is at least one fixed point of this mapping. As in Eisenberg and Noe (2001), we will find it by the method of successive approximation, which these authors call the 'fictitious default' algorithm.

Besides establishing existence, Eisenberg and Noe (2001) also provided conditions under which the fixed point of the mapping is unique for the case of zero liquidation costs. We instead find a robust set of examples with positive liquidation costs in which there are at least two fixed points. We first provide an example and then an interpretation of the two fixed points as alternative bankruptcy regimes.

Example 1 (Non Uniqueness of Clearing Vectors)

Consider the case of three banks that have ex-post asset values, $\tilde{A}_i$ of $1.011, 0.972$, and 1.048, for i = 1,2, and 3, respectively. Each bank has deposits of 1. Bank 1 owes banks 2 and 3 0.403 each, bank 2 owes 0.391 each, and bank 3 owes 0.414 each. Proportional liquidation costs $\Phi = 0.1$. Then there are two clearing vectors. In the first, the payments made by the banks to the other banks are $\{0.806, 0.782, 0.828\}$, that is, each bank makes a full payment. It is easily verified that with these payments, each bank is able to make its full payments to all depositors and banks, and is not liquidated. For example, bank 2 receives $0.403 + 0.414 = 0.817$, so that its total resources available for distribution are $0.972 + 0.817 = 1.789$. Its total commitments are 1.782, so it does not face liquidation.

The other clearing payment vector is $\{0, 0, 0\}$, that is no bank makes any payments to other banks or receives anything. Now all three banks are insolvent, and their assets are liquidated. For example, consider the first bank. Since it receives nothing, it has total assets of 1.011 and commitments of 1.806, hence it is liquidated. After liquidation, it has $0.9 \cdot 1.011$ available for distribution, which equals 0.9099. This is clearly smaller than the 1 it owes its depositors. So, its pays 0 for all its interbank commitments. The same happens to the other banks. Notice the "systemic" risk in this clearing payment vectors. Each bank defaults on its commitments only because it receives nothing on commitments owed to it. We will make this definition more precise below.

The role of a non-zero Φ is important. If $\Phi = 0$ then as in Eisenberg and Noe (2001), we would have a single clearing vector, the first one. For any $\Phi > 0.01$ though, the second clearing vector is also valid. Finally, its worth pointing out that the example is a little extreme because with the second clearing vector all payment vectors are zero. We can construct similar examples where only one or two banks have zero clearing payments.

Motivated by the example and following Elsinger, Lehar, and Summer (2006) we make a distinction between 'fundamental" defaults, and "contagious" defaults. The default of bank i is called fundamental if bank i is not able to honor its promises under the assumptions that all other banks honor their promises,

$$\sum_{j=1}^{F} \tilde{\Pi}_{ji}^{\mathcal{F}} d^{\mathcal{F}}(\{j\}) + \tilde{e}_i^{\mathcal{F}} - d^{\mathcal{F}}(\{i\}) < 0. \tag{4}$$

A contagious default occurs, when bank i defaults only because other banks are not able to keep their promises, i.e.,

$$\sum_{j=1}^{F} \tilde{\Pi}_{ji}^{\mathcal{F}} d^{\mathcal{F}}(\{j\}) + \tilde{e}_i^{\mathcal{F}} - d^{\mathcal{F}}(\{i\}) \quad \geq \quad 0 \tag{5}$$

$$\text{but} \tag{6}$$

$$\sum_{j=1}^{F} \tilde{\Pi}_{ji}^{\mathcal{F}} p^{\mathcal{F}}(\{j\}) + \tilde{e}_i^{\mathcal{F}} - d^{\mathcal{F}}(\{i\}) \quad < \quad 0. \tag{7}$$

Using these definitions, the defaults in the second clearing vector are all contagious.

We interpret the two different clearing vectors as two distinct bankruptcy regimes. The first we call the "strong" regime, since it implies that all banks pay larger amounts for their interbank commitments, and in turn receive more from other banks. The second is the "weak" regime, in which banks pay out less and receive less on their commitments. Both clearing vectors are 'fair' in the sense that limited liability of all equity holders and absolute priority of all claims is maintained in both. The choice of the regime is determined by the enforcement power of the regulator, and its determination is outside the scope of this model. However, we note that unlike the analysis in Eisenberg and Noe (2001) and Elsinger, Lehar, and Summer (2006), we do not assume that banks actual payments for their interbank claims are determined completely by the clearing payment vectors. The clearing vector is the value that each bank will pay out if the set of banks jointly fail to renegotiate all claims among themselves, and approach the regulator. In the next section we model these renegotiations and then study the implications for recovery rates on the interbank claims in the two bankruptcy regimes.

2 Renegotiation of Interbank OTCD Payments

In this section we provide an analysis of the bargaining game that takes place at date 1 between the N banks if some or all of them fail to make full payments on the interbank OTCD commitments. The banks consider mergers with each other for the resolution of their claims.

2.1 Externalities and Games in Partition Form

In considering the strategies of banks if they decide to merge, we must consider the value that can be realized by a coalition of banks. Games that start with the specification of values of coalitions are called characteristic function games. In such games a coalition S of the set of banks can obtain the payoff $v(S)$ irrespective of the actions of other banks.[10] In contrast, in this paper the value that a coalition S can obtain depends on the actions and the merging strategies of other banks. Let the merging strategies of the full set of banks lead to a partition $\mathcal{F}$ of the set of all banks. Then, we will write the value of a coalition of banks S as $v^{\mathcal{F}}(S)$ to denote the value that this set of banks can attain when playing in the partition $\mathcal{F}$. In general the value will be different when different partitions are formed. In particular we are interested in what happens to the value of S when two

[10] Probably the most famous solution of characteristic function games is the Shapley value, which assigns the average marginal product to each bank and satisfies very reasonable axioms of a bargaining process. For games with a well defined characteristic function, Hart and Mas-Colell (1996) provides an explicit non-cooperative alternating offers game in which the N banks agree to assign each bank her Shapley value. It will be evident from our analysis that the solution of our game will be the Shapley value when there are no externalities so that the endogenously chosen partition by the banks is the grand coalition, but will differ otherwise.

banks in the partition $\mathcal{F}$ merge. To be formal, let F_1 and F_2 denote two coalitions in the partition $\mathcal{F}$. Let $\mathcal{F}_{12}$ denote the partition that is formed when the two banks F_1 and F_2 in $\mathcal{F}$ merge, and all other banks remain the same. Following Maskin (2003) we say that the externality from the merger is positive if for some coalition of banks $F \notin (F_1 \bigcup F_2)$ in $\mathcal{F}$,

$$v^{\mathcal{F}}(F) < v^{\mathcal{F}_{12}}(F). \tag{8}$$

Therefore, the value of the coalition F increases when coalitions F_1 and F_2 merge.

Such positive externalities naturally arise in financial applications. Think of an insolvent borrower with two well capitalized creditors. By merging with the insolvent borrower one creditor can bail it out and thus reduce potential liquidation costs. The merger however creates a positive externality for the other creditor, who can collect its full promised payment. The value that this second lender can achieve clearly depends on the strategy of the borrower and the first lender.[11] The existence of positive externalities implies that an efficient outcome will not always be achieved. In the example above, one bank has an incentive to free ride and hope that the other bank will bail out the failed institution. We will show more detailed examples of an inefficient outcome later after we have defined the rules of the bargaining game.

2.2 The Bargaining Protocol

The game starts with nature choosing an order of the banks at which they will arrive at the bargaining site. The order is maintained in two important stages of the game: (i) A bank higher in the order gets to bid for the claims of all banks lower in the order, and (ii) If two banks remain independent, and they both bid for a third bank, then a bank higher in the order places the earlier bid. Conditional on the randomly picked order, banks make take-it-or-leave-it offers, that is the bank receiving the order can simply reject the bid or fail to reject it and compare the bid with competing bids from other banks.[12]

The game proceeds as follows: The first bank forms a singleton coalition. Each later bank n that arrives, faces a partition, i.e. a collection of coalitions $1, \cdots, F$. Coalition 1 makes a bid for bank n, which is an offer of a cash payment that n gets in exchange for signing up with the

[11] Externalities are present in a lot of economic problems where coalitions are formed. Consider, for example, cartels, where the price increase due to output reduction of the cartel members benefits firms that choose not to join the cartel. For an excellent summary of the recent literature see Ray (2007).

[12] Our choice of these sequential rules of bargaining is motivated by the work of Maskin (2003) who shows that with such rules there is a larger set of circumstances in which banks will find a bargaining solution relative to a game where banks can make simultaneous offers. See in particular Example 1 of his paper.

coalition and surrendering all its claims. Bank n either rejects the offer (in which case we assume that coalition 1 and bank n can never be in the same coalition again), or it fails to reject the bid, and then it can entertain bids from coalitions $2 \cdots F$, and pick the highest bid. As we will see, the optimal bidding strategy of the bank with the highest valuation is to bid the maximum of bids of the other banks' valuations. Joining a coalition is a binding merger agreement, i.e., bank cannot leave a coalition at a later stage of the game. If bank n rejects all bids, it will remain independent, i.e. it will be in a singleton coalition. Once all banks are assigned to coalitions, the final partition is determined and payoffs for coalitions are realized. The banks who joined a coalition receive the payoff they were promised upon signing up, and the payers that started a coalition keep the payoff of the coalition minus the payments that they promised to the other coalition members. Since the set of bank mergers and hence the resulting eventual partition of banks is endogenous, we will keep a track of both banks' payoffs (by the function ϕ) and the partition (by the function ψ) at each node of the tree. We will say that *renegotiations break down* whenever the partition of the game reached optimally in the bargaining game does not lead to an efficient liquidation policy, which we will characterize below.

The clearing vector of the economy determines the reservation values for banks as they evaluate bids to be taken over. If there are no merger agreements each bank can obtain a minimum payoff of $v^{\mathcal{N}}(\{i\})$ that satisfies

$$v^{\mathcal{N}}(\{i\}) = \max[\tilde{e}_i + r^{\mathcal{N}}(\{i\}) - \tilde{d}^{\mathcal{N}}(\{i\}), 0]. \tag{9}$$

Note that $v^{\mathcal{N}}\{i\})$ is completely determined by the contractual ex-post payment the bank is obliged to make and the clearing vector that is enforced by the bankruptcy regime. Similarly, we can define the minimal threat points of various subsets of banks, when they are in a game with a partition $\mathcal{F}$. Then the payoff that the subset $S \in \mathcal{F}$ obtains without renegotiation is

$$v^{\mathcal{F}}(S) = \max[\sum_{i \in S} \tilde{e}_i + r^{\mathcal{F}}(S) - \tilde{d}^{\mathcal{F}}(S), 0]. \tag{10}$$

To make the definition of efficiency more precise we define another function $w(S)$ that is the maximum amount that the banks in the coalition S can obtain using the given set of interbank securities and an optimal merger and liquidation policy by the members of S. For example for a two bank game,

$$w(\{1,2\}) = \max\left(v^{\{1,2\}}(\{1,2\}), v^{\{\{1\},\{2\}\}}(\{1\}) + v^{\{\{1\},\{2\}\}}(\{2\})\right), \tag{11}$$

The first term is the value that the two banks realize if they merge, while the second term arises if they do not merge and either of them is possibly liquidated. We will say the liquidation policy in a 2 bank game is efficient if $w(\{1,2\}) = \phi^{\psi^*}(\{1\}) + \phi^{\psi^*}(\{2\})$, where ψ^* is the filtration formed by the optimal action of the banks in the bargaining game, that is, ψ^* equals $\{1,2\}$ if the banks decide to merge, or $\{\{1\},\{2\}\}$ if the banks decide to stay independent.

2.3 Solving for Equilibrium of the Two Bank Case

We first provide an analysis for the two bank case where as we see the equilibrium of the bargaining game always leads to an ex-post efficient liquidation policy. The analysis could be for an economy with only two banks, or for a subgame with two merged banks from a larger banking system. We simply write all payoffs conditional on the current filtration being $\mathcal{F} = \{1,2\}$, which could arise from either of both these situations. For the two bank case, the clearing payment vector in (2) can be written more simply as

$$p^{\mathcal{F}}(\{i\}) = \min\left[\tilde{l}_{ij}^{\mathcal{F}}, \max\left(A_i^{\mathcal{F}} - \Phi A_i^{\mathcal{F}} \mathbf{1}_{p^{\mathcal{F}}(\{i\})<d_i^{\mathcal{F}}} - L_i^{\mathcal{F}} + p^{\mathcal{F}}(\{j\}), 0\right)\right], \tag{12}$$

for i = 1,2, and $j \neq i$. and the clearing receiving vector is $\{r^{\mathcal{F}}(\{1\}), r^{\mathcal{F}}(\{2\})\} = \{p^{\mathcal{F}}(\{2\}), p^{\mathcal{F}}(\{1\})\}$, since all the payments made by bank i are received by bank j. We now provide simple conditions under which it is optimal (efficient) to liquidate one or more banks.

Result 1 *In the two person game the necessary and sufficient efficient conditions under which at least one bank is liquidated are as follows: Either $\tilde{e}_1^{\mathcal{F}} + \tilde{e}_2^{\mathcal{F}} < 0$ or $\tilde{e}_1^{\mathcal{F}} + \tilde{e}_2^{\mathcal{F}} > 0$ and for $i \neq j$*

$$\tilde{e}_i^{\mathcal{F}} + d^{\mathcal{F}}(\{j\}) < 0 \quad \textit{and} \tag{13}$$

$$\tilde{e}_j^{\mathcal{F}} - d^{\mathcal{F}}(\{j\}) > 0, \tag{14}$$

in which case bank j will force a liquidation of the assets of bank i. Apart from these two cases, $v^{\mathcal{F}}(\{1,2\}) = \tilde{e}_1^{\mathcal{F}} + \tilde{e}_2^{\mathcal{F}}$.

The optimal liquidation policy ensures that the value of combining the claims of two banks cannot be smaller than the value of each bank alone. If one of the banks has equity so far negative that even after receiving its interbank contractual clearing payment, its value is still negative, then the two bank coalition will optimally liquidate this bank, and share the resources of the other bank.

Result 2 *The two bank bargaining game always leads to efficient liquidations as in Lemma 1. The payoffs of the two banks are obtained from averaging over the bidding orders that follow below:*

(i) If $p^{\mathcal{F}}(\{i\}) = d^{\mathcal{F}}(\{i\})$ for $i = 1, 2$, the order of bidding is irrelevant and each bank gets $\tilde{e}_i + d_j - d_i$.

(ii) If $p^{\mathcal{F}}(\{i\}) = 0$, and $p^{\mathcal{F}}(\{j\}) = d_j$, the order of bidding is irrelevant, bank i is liquidated, and bank j obtains $\tilde{e}_j - d_j$.

(iii) If $0 < p^{\mathcal{F}}(\{i\}) < d_i$, and $p^{\mathcal{F}}(\{j\}) = d_j$, then neither bank is liquidated. Bank i obtains $\tilde{e}_i - p^{\mathcal{F}}(\{i\}) + d_j$ and bank j gets $\tilde{e}_j + p^{\mathcal{F}}(\{i\}) - d_j$ if bank i bids first. If bank j bids first, then it gets $\tilde{e}_i + \tilde{e}_j$ and bank i gets 0.

(iv) If $0 < p^{\mathcal{F}}(\{i\}) < d_i$ for $i = 1, 2$, then the first bidder obtains $\tilde{e}_i + \tilde{e}_j$ and the second bidder gets 0.

The intuition for the bargaining equilibrium leading to an efficient liquidation policy is that if the solvent bank decides to bail out the bank in trouble, it can fully appropriate the preempted liquidation costs. We shall see in the following subsections when there are three banks this result will no longer hold. It is also useful to note that for the case that both banks remain solvent, there is no benefit to the banks from merging since the reservation values of the banks equal their values in solvency. Thus our model has no implications for bank mergers over and above their role in resolving financial distress.

We now illustrate with a simple example why replacing the condition (14) by the weaker one: $\tilde{e}_j^{\mathcal{F}} - p^{\mathcal{F}}(\{j\}) > 0$ will not provide a sufficient condition for liquidating the insolvent bank.

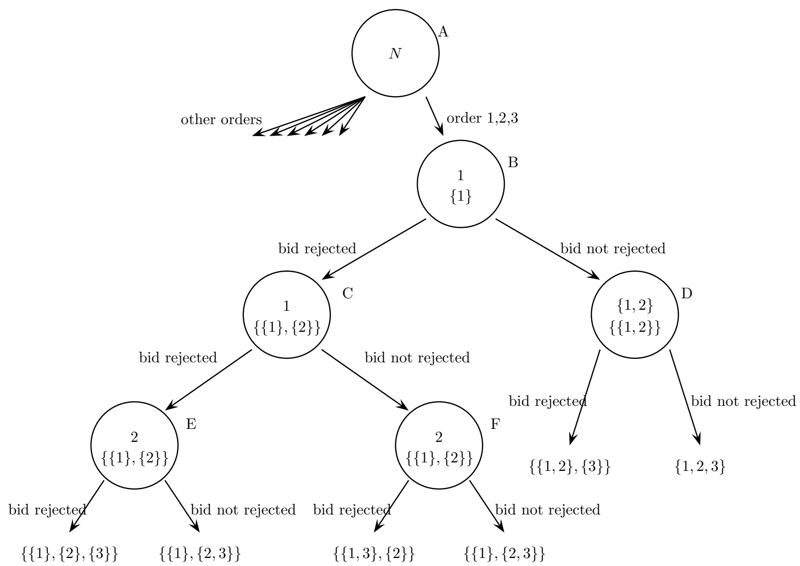
Example 2:

Let $A_1 = 2$, $A_2 = 0.7$, $L_i = 1$, and $l_{ij} = 1.2$ for $i, j = 1, 2$. Let $\Phi = 0.4$. Then bank 1 makes a payment of $p_1 = 0.6 \cdot 2 - 1 = 0.2$, while bank 2 pays 0 on its OTCD commitments. Then $e_1 - p_1 = 0.8 > 0$, but if bank 1 forces a liquidation of bank 2, its equity holders keeps a profit of 0 for themselves, since all its assets are exhausted meeting the OTCD commitments. Merging the banks however gives the combined shareholders positive equity of 0.7, which by Result 2 can all be appropriated by the equityholders of bank 1.

2.4 Solving for Equilibrium of the Three Bank Case

In the three bank case we can analyze the game in greater detail. Figure 3 shows the extensive form of the three bank game. Expected payoffs are obtained by averaging over the orders. Denote by b_i^X

Figure 3: Extensive form of the three bank game



Each node has the active bank (first line) and the partition, which is realized (second line). First, at node A, Nature chooses an order in which banks arrive at the bargaining site. The figure illustrates the game for the natural order 1,2,3. At node B, bank 1 makes a bid for bank 2. If the bid is not rejected, banks 1 and 2 merge (node D) and the merged bank can then make a bid for bank 3. If bank 3 accepts this bid, the grand coalition of banks forms, otherwise it remains independent. If bank 1's bid for bank 2 is rejected, then the two banks remain independent and are both potential acquirers of bank 3 (node C). Bank 1 bids first and if its bid is rejected by bank 3 the game moves to node E where bank 2 can make a bid for bank 3. If bank 1's bid is not rejected by bank 3, then the game moves to node F where bank 2 can make an additional bid for bank 3. Bank 3 chooses the higher of the bids.

the bid that bank i makes at node X and let $\bar{b}_i^X$ be the maximum that bank i is willing to bid at node X.

Result 3 *The solution of the three bank bargaining game for the natural arrival order, i.e.,* $1, 2, 3$ *given in Figure 3 is as follows:*

(i) Conditional on the game reaching node E, bank 2's maximum bid $\bar{b}_2^E$ *is*

$$\bar{b}_2^E = v^{\{1\},\{2,3\}}(\{2,3\}) - v^{\mathcal{N}}(\{2\}). \tag{15}$$

(a) If $\bar{b}_2^E < v^{\mathcal{N}}(\{3\})$, *then* $b_2^E = \bar{b}_2^E$,
$\phi^E = \left(v^{\mathcal{N}}(\{1\}), v^{\mathcal{N}}(\{2\}), v^{\mathcal{N}}(\{3\})\right)$ *and the realized partition is* $\psi^E = \mathcal{N} = \{\{1\},\{2\},\{3\}\}$,

(b) Otherwise, $b_2^E = v^{\mathcal{N}}(\{3\})$, $\phi^E = \left(v^{\{\{1\},\{2,3\}\}}(\{1\}), v^{\{\{1\},\{2,3\}\}}(\{2,3\}) - v^{\mathcal{N}}(\{3\}), v^{\mathcal{N}}(\{3\})\right)$ *and the realized partition is* $\psi^E = \{\{1\},\{2,3\}\}$.

(ii) At node C, the winning bidder must make a bid of at least $\phi^E(\{3\})$ for bank 3, otherwise it will be rejected. Bank 1's maximum bid for bank 3 is

$$\bar{b}_1^C = \max(v^{\{1,3\},\{2\}}(\{1,3\}) - v^{\{2,3\},\{1\}}(\{1\}), \phi^E(\{3\})), \tag{16}$$

and if bank 1's bid is rejected, the game moves to node F and bank 2's maximum bid is

$$\bar{b}_2^F = \max(v^{\{1\},\{2,3\}}(\{2,3\}) - v^{\{\{1,3\},\{2\}\}}(\{2\}), \phi^E(\{3\})). \tag{17}$$

At node F, the payoffs and realized partitions are

- *(a) $\bar{b}_2^F > \bar{b}_1^C$, then $\psi^F = \{\{1\},\{2,3\}\}$, and $\phi^F = \{v^{\{1\},\{2,3\}}(\{1\}), v^{\{1\},\{2,3\}}(\{2,3\}) - \bar{b}_1^C, \bar{b}_1^C\}$,*
- *(b) Otherwise, $\psi^F = \{\{2\},\{1,3\}\}$, and $\phi^F = \{v^{\{1,3\},\{2\}}(\{1,3\}) - \bar{b}_2^F, v^{\{1,3\},\{2\}}(\{2\}), \bar{b}_2^F\}$.*

(iii) At node C,

- *(a) If $\phi^E(\{1\}) > \phi^F(\{1\})$, then $\psi^C = \psi^E$ and $\phi^C = \phi^E$.*
- *(b) Otherwise, $\psi^C = \psi^F$ and $\phi^C = \phi^F$.*

(iv) Conditional on the game reaching node D, bank 1's maximum bid for bank 3 satisfies:

$$\bar{b}_1^D = v^{\{1,2,3\}}(\{1,2,3\}) - v^{\{1,2\},\{3\}}(\{1,2\}), \tag{18}$$

and the realized payoffs and partitions are

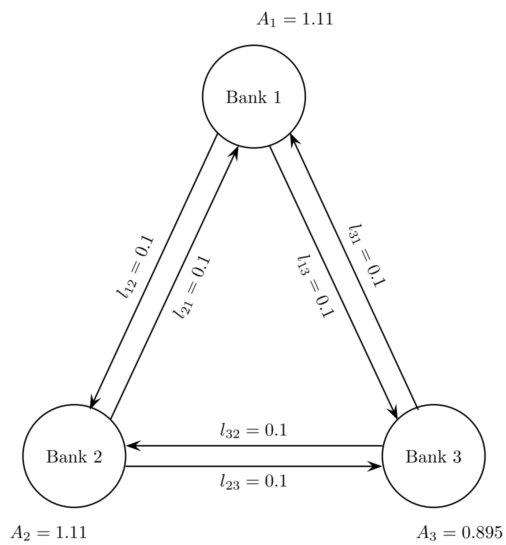
- *(a) If $\bar{b}_1^D > v^{\{1,2\},\{3\}}(\{3\})$, $\phi^D = \left(v^{\{1,2\},\{3\}}(\{1,2\}) - b_1^B, b_1^B, v^{\{1,2\},\{3\}}(\{3\})\right)$, and the realized partition is $\psi^D = \{\{1,2\},\{3\}\}$,*
- *(b) Otherwise, $\phi^D = \left(v^{\{1,2,3\}}(\{1,2,3\}) - b_1^B - v^{\{1,2\},\{3\}}(\{3\}), b_1^B, v^{\{1,2\},\{3\}}(\{3\})\right)$, and the grand coalition is realized $\psi^D = \{1,2,3\}$.*

(v) At node B, the node where bank 1 first makes a decision:

- *(a) If $\phi^D(\{1\}) > \phi^C(\{1\})$ $b_1^B = \phi_2^C$ $\phi^B = \phi^D$, and the partition $\psi^B = \psi^D$ as defined in (iv) will be realized,*
- *(b) Otherwise $\phi^B = \phi^C$, and the partition $\psi^B = \psi^C$ as defined in (iii) will be realized.*

The ex ante payoffs for the banks ϕ are determined by averaging ϕ^B over all possible arrival orders.

Figure 4: Example to Illustrate Inefficient Liquidation.



2.4.1 Example: Renegotiation Inefficiency

To see how externalities can cause inefficient liquidations, consider the example illustrated in Figure 4.The outside liabilities L for each bank are assumed to be 1 and proportional liquidation costs are $\Phi = 0.3$. Bank 3 is in fundamental default. Even when it can collect all the promised payments from the other banks of 0.2, it cannot meet its interbank obligations and has net value of $A_1 - L + l_{13} + l_{23} - l_{31} - l_{32} = 0.895 - 1 + 0.1 + 0.1 - 0.1 - 0.1 = -0.105$. Thus, if no renegotiations occur, bank 3 is liquidated, reducing its asset value to $A_3(1 - \Phi) = 0.895(1 - 0.3) = 0.6265$. Even when bank 3 collects all interbank claims, it has $0.6265 + 0.2 = 0.8265$ which is less that its outside liabilities L and bank 3 shareholders as well as the other banks receive zero. Banks 1 and 2 are well enough capitalized to survive.

For each possible partition we can then compute the clearing vector according to Equation (3) and the value of the equity holders' claim without renegotiations using Equation (10).

Coalition structure	Payoff equityholders	Comments
{1,2,3}	$v^{\{1,2,3\}}(\{1,2,3\}) =$ 0.115	$A_1 + A_2 + A_3 - 3L$
{1,2},{3}	$v^{\{1,2\},\{3\}}(\{1,2\}) =$ 0.02	
	$v^{\{1,2\},\{3\}}(\{3\}) =$ 0	Bank 3 fails
{1,3},{2}	$v^{\{1,3\},\{2\}}(\{1,3\}) =$ 0.005	
	$v^{\{1,3\},\{2\}}(\{2\}) =$ 0.11	
{1},{2,3}	$v^{\{1\},\{2,3\}}(\{1\}) =$ 0.11	
	$v^{\{1\},\{2,3\}}(\{2,3\}) =$ 0.005	
$\mathcal{N}$={1},{2},{3}	$v^{\mathcal{N}}(\{1\}) =$ 0.01	
	$v^{\mathcal{N}}(\{2\}) =$ 0.01	
	$v^{\mathcal{N}}(\{3\}) =$ 0	Bank 3 fails

There are three possible partitions that maximize welfare, i.e. the sum of the banks' payoffs: the grand coalition $\{1,2,3\}$ and the two partitions in which one of the financially sound banks bails out the troubled bank $\{1,3\}, \{2\}$ and $\{1\}$,{2,3}. As we will see below, the welfare maximizing outcome cannot be realized in all cases, because an individual bank is better off liquidating a troubled bank than bailing it out. If Bank 1 bails out bank 3, it can at most get $v^{\{1,3\},\{2\}}(\{1,3\}) = 0.005$, whereas it can get $v^{\mathcal{N}}(\{1\}) = 0.01$ if bank 1 is liquidated.

In the discussion we follow the extensive form and refer to nodes as labeled in Figure 3. Which partition will be realized depends on the order of arrival. Consider the representative cases of arrival orders:

Arrival order 1 2 3: Consider the subgame in node E first: Banks 1 and 2 are separate and Bank 3 has rejected Bank 1's bid. Bank 2 has to decide how much to bid for Bank 3. By staying independent Bank 2 will get $v^{\mathcal{N}}(2) = 0.01$ whereas it could get at most $v^{\{1\},\{2,3\}}(\{2,3\}) = 0.005$ from signing up 3. Bank 2 will therefore make an infeasible bid for Bank 3 which will get rejected and the realized partition is $\psi^E = \mathcal{N}$.

Next consider the Subgame in Node F. If Bank 1 has made an acceptable bid, Bank 2 is happy to stay independent and collecting $v^{\{1,3\},\{2\}}(\{2\}) = 0.11$ rather than merging with 3 and getting at most 0.005. In node C, Bank 1 anticipates that it will not get a competitive bid from Bank 2. Thus by making any acceptable bid for Bank 3, Bank 1 will sign up 3 and collect at most $v^{\{1,3\},\{2\}}(\{1,3\}) = 0.005$. It is better for Bank 1 to make an unacceptable offer to bank 3, and let the game continue to node E, where $\psi^E = \mathcal{N}$ is realized, and Bank 1 will collect $v^{\mathcal{N}}(1) = 0.01$. Thus we know that whenever the game comes to node C, the partition $\psi^C = \mathcal{N}$ will be realized.

Intuitively, if Banks 1 and 2 cannot agree to cooperate the game reaches node C and coordination failure emerges. Even though it would be welfare increasing, neither bank wants to bail out 3 because it would make them individually worse off. An inefficient outcome, where each bank stays independent is realized. Specifically Banks 1 and 2 get a payoff of $v^{\mathcal{N}}(1) = v^{\mathcal{N}}(2) = 0.01$ in the subgame starting at C.

In node B Bank 1 therefore can sign up bank 2 for any price just above 0.01. Is this worthwhile for Bank 1? When signing up 1, the game continues to node D, where Bank 1 considers signing up 3 as well. By staying independent, bank 3 can realize a value of zero. Therefore Bank 1 can sign up 3 for 0 and keep for itself $v^{\{1,2,3\}}(\{1,2,3\}) = 0.115$ minus the cost of signing up 2 and 3, that is 0.115-0.01-0=0.105.

Thus Bank 1 can realize a value of 0.105 by signing up 2 and continuing to node D, or get 0.01 by making an infeasible bid for Bank 2 and moving on to node C. Bank 1 will optimally chose the former strategy and the grand coalition will be realized. The individual banks realize payoffs of 0.105,0.01, and 0, respectively.

Intuitively Banks 1 and 2 anticipate that staying independent leads to an inefficient outcome. Acting jointly eliminates the possibility for each bank to free ride on the other and thus prevents coordination failure. This efficient equilibrium is only supported by two arrival orders: 1,2,3 and 2,1,3. These orders allow the two solvent banks to coordinate their actions before the troubled bank arrives at the bargaining site. By merging before bank 3 arrives, banks 1 and 2 can effectively reduce the three bank game to a two bank game, which always has an efficient solution.

Arrival order 1 3 2: Start again in node E. Banks 1 and 3 do not cooperate, bank 2 has rejected Bank 1's bid and it is now Bank 3's turn to extend an offer to Bank 2. By staying independent, Bank 2 can realize a payoff of $v^{\mathcal{N}}(\{2\}) = 0.01$. The most that 3 could bid for Bank 2 is $v^{\{\{1\},\{2,3\}\}} = 0.005$. Thus in node E, bank 2 will not bail out 3, every bank stays independent, and bank 3 fails.

In node F, Bank 1 has made a feasible bid for 2. Bank 2 will accept any bid that is at least as high as what Bank 2 gets by rejecting and ending up in node E. Thus Bank 1 has made a bid of at least 0.01. Bank 3 cannot offer more than $v^{\{\{1\},\{2,3\}\}}(\{2,3\}) = 0.005$ and therefore Bank 1 will win 2 and Bank 3 fails. In node C Bank 1 is indifferent between signing up 2 or not. By signing up 2, the game moves to node F and bank 1 will get $v^{\{\{1,2\},\{3\}\}} - 0.01 = 0.01$. By making an unacceptable bid for 2, the game continues to node E and Bank 1 collects $v^{\mathcal{N}}(\{1\}) = 0.01$. Similar to the previous case we see that when Banks 1 and 3 do not merge, coordination failure arises and an inefficient solution is realized.

To analyze node D suppose that banks 1 and 3 have merged, i.e. Bank 1 is bailing out bank 3. Then Bank 2 has a strong incentive to free ride and not contribute to the rescue of Bank 3. Formally, by staying independent Bank 2 can realize $v^{\{\{1,3\},\{2\}\}}(\{2\}) = 0.11$. Bank 2 will not accept any offer below that and when Bank 1 sign up 2 there is only $v^{\{1,2,3\}}(\{1,2,3\}) - 0.11 = 0.005$ left for banks 1 and 3 to share. Banks 1 and 3 can share the same amount if they do not sign up bank 2. At node B, bank 1 can therefore make at most 0.005 by signing up 3 which is less than the 0.01 that Bank 1 can get by making an unacceptable offer to 3 and continuing in node C. Thus, the overall outcome is that of the subgame starting in C in which each bank stays independent and bank 3 fails. The payoffs for banks 1,2, and 3 are 0.01,0.01,and 0, respectively. The solution is inefficient as the sum of the payoffs is less than what could have been achieved by the grand coalition.

The inefficiency arises whenever a solvent bank moves last. If the other solvent bank has agreed to bail out Bank 3, the last mover can stay independent and collect its full interbank payments. The other solvent bank anticipates that it will have to carry the burden of bailing out 3 alone and thus will optimally decide to stay independent. If the first solvent bank does not bail out Bank 3, the last mover has no incentive to do so either and Bank 3 fails. In this example four out of six possible arrival orders lead to an inefficient outcome. The following table summarizes the payoffs for all orders of proposers:

Order proposers	Bank 1	Bank 2	Bank 3	realized partition
1 2 3	0.105	0.01	0	$\{1,2,3\}$
1 3 2	0.01	0.01	0	$\mathcal{N}$
2 1 3	0.01	0.105	0	$\{1,2,3\}$
2 3 1	0.01	0.01	0	$\mathcal{N}$
3 1 2	0.01	0.01	0	$\mathcal{N}$
3 2 1	0.01	0.01	0	$\mathcal{N}$
Average	0.0258	0.0258	0	

The coordination problem that creditors face in some arrival orders and the resulting inefficiency can be potentially resolved by regulatory intervention. Our model provides a possible explanation for recent cases like AIG or Bear Sterns, where bank regulators intervened and coordinated the bailout renegotiations.

2.5 The Failure of the Coase Theorem and an Analogy to Bank Runs

Ronald Coase, who won a Nobel prize for his work on externalities, argued that, so long as property rights are clearly established, externalities will not cause an inefficient allocation of resources as long as there are no "transactions costs".[13] These transactions costs are the costs for enforcing contracts when there are incentive problems. Modern writers (see, e.g., Tirole (1988)) also assert that the bargaining outcomes are ex-post efficient as long as information is perfect. This result in called the Coase theorem. In contrast, we find in the previous subsection that the bargaining solution is ex-post inefficient, in violation of the Coase Theorem as the positive externalities caused by the bailout of the insolvent banks by one of the solvent banks lead to a renegotiation breakdown, and avoidable liquidation costs are incurred.

In our setting, as advocated by proponents of the Coase theorem, banks enter into OTCD contracts to share risks with each other. The contracts imply that stronger banks make payments to weaker banks ex-post. However, banks do not risk share perfectly since it reduces their incentives to maintain the quality of their underlying streams. Thus there is some residual unhedged risk in the financial system. The conflict arises when one bank is insolvent and two are not. In this case, while the two solvent banks may jointly agree that the insolvent bank should be bailed out, and hence its liquidation costs avoided, they disagree on *who* should bear the costs of the bailout. Each solvent bank therefore "runs" from the system by refusing to carry out the bailout since all the benefits from carrying out the bailout accrue to the other banks. Therefore, the inefficiency in the system is a coordination failure among the two solvent banks. In this sense our equilibrium, in the context of a bank threatening to withdraw its resources from the banking system, is similar to deposits withdrawing their deposits from a failing bank, and hence our equilibrium represents a "system run" by solvent banks.

We also find it relevant to point out important differences of this coordination failure from the bank run problem in Diamond and Dybvig (1983). The bank run in their model is dependent on a queuing structure for deposits, and is a self-fulfilling prophecy: if all depositors do not run to the

[13] We take a quote from the economist magazine that explains the property rights argument:

> Markets find ways to take account of externalities - ways to "internalize" them, as economists say, more often than one might think. Bees are to externalities as lighthouses are to public goods. For years they served as a favorite textbook example. Bee-keepers are not rewarded for the pollination services they provide to nearby plant-growers, so they and their bees must be inefficiently few in number. Again, however, the world proved cleverer than the textbooks. Cheung (1973) studied the apple-growers of Washington state and discovered a long history of contracts between growers and beekeepers. The supposed market failure had been effectively - and privately - dealt with. (The Economist, February 17th 1996, p.67)

bank, the bank will remain solvent, and vice versa. As in their model, solvent banks run from the system rather than bail out the troubled bank. However, unlike the depositors who each behave myopically in the Diamond and Dybvig (1983) model, we explicitly allow for banks to possibly merge and have a coordinated policy of bailing out the insolvent bank. However, merging is not incentive compatible, and hence is not carried out in equilibrium.

3 Optimal Effort and OTCD Contract Choices by Banks

In our model, banks must expend effort to improve the quality (mean) of their outside asset streams in Assumption 1. They also participate in the interbank market for risk sharing using the OTCDs in Assumption 3. The banks have an incentive to reduce the variance of their equity value even though they are risk neutral. A reduction in risk decreases the bank's default probability and thus liquidation costs, which are borne by its equity holders ex ante. The OTCDs have three components: pure interbank loans, asset swaps, and credit default swaps, each of which can potentially help share the risk in banks' asset streams, but by effectively transferring the rights of these asset streams reduce the incentive of banks to maintain the quality of their asset streams (the moral hazard problem). In addition, the three components have different impacts on the ability of stronger banks to run from the system and hence avoid the bailout costs of weaker banks leading to inefficient liquidations.

In this section, we study the optimal choice of interbank OTCD contracts in this setting, under the assumption that banks either renegotiate their OTCDs ex-post and maximize their bargaining values as in Section 2, or do not renegotiate and maximize their profits obtain from the clearing payment vector determined by the regulator as in Section 1.1. We refer to the two cases as "with renegotiations" and "with the clearing vector," respectively. One reason for formulating renegotiations is that it potentially mitigates the moral hazard problem, since the benefits of the effort are better captured by the bank in renegotiations, in which it is able to extract greater value from the other banks in the system. In addition, we will see the recovery processes under the two sets of assumptions change the properties of the hedging components and affects their effectiveness in bankruptcy cost reduction, providing incentives, and preventing system runs.

3.1 The Banks and Social Planner's Optimization Problems

First consider the case without renegotiations. Then, we can write bank i's ex ante profit as

$$\pi^{CV}(\{i\}) = \text{Max}_{a,b,c}\left[\text{Max}_{h_i(a,b,c)}\left(E[v^{\mathcal{N}}(\{i\}) - \omega^{D,CV}(\{i\}) - \gamma \cdot h_i^2]\right)\right], \tag{19}$$

where $v^{\mathcal{N}}(\{i\})$ is ex-post profit of bank i determined by the clearing vector shown (9), $\omega^{D,CV}(\{i\})$ is the deposit insurance premium in Assumption 2, and the expectation is taken over all realizations of the asset values, $\tilde{A}_i$ in Assumption 1. For convenience, we solve the problem in two stages, first choosing the terms of the interbank contracts (a, b, c) and then choosing the level of effort h_i conditional on the contracts. This follows since banks asset values have identical distributions and are specified as *quid pro quo* exchanges so that the contract choices are by definition common for all firms. We solve for individual effort choices, which also turn out to be equal across banks due to symmetry. Notice that the effort choice has an externality since the OTCDs partly transfer the benefits of the improved asset stream to increasing the profits and lowering liquidation costs at other banks. Bank i can appropriate these benefits only to the extent that it obtains better recoveries when these other banks have low asset realizations. Therefore, as in any public goods problem, bank i chooses an effort level that maximizes only its personal profit, which is generally lower that the socially optimal level. The socially optimal choice of effort maximizes

$$\pi^{CV} = \text{Max}_{a,b,c}\left[\text{Max}_{h_i(a,b,k)=h(a,b,c)}\left(\sum_{i=1}^{N} E[v^{\mathcal{N}}(\{i\}) - \omega^{D,CV}(\{i\}) - \gamma \cdot h^2]\right)\right], \tag{20}$$

where the term $h_i = h$ denotes the constraint that each bank makes the same effort choice, and this level is chosen to maximize the joint profits of the banks in the system.

Similarly, we formulate the individual bank's problems with renegotiations as

$$\pi^{R}(\{i\}) = \text{Max}_{a,b,c}\left[\text{Max}_{h_i(a,b,c)}\left(E[\phi(\{i\}) - \omega^{D,R}(\{i\}) - \gamma \cdot h_i^2]\right)\right], \tag{21}$$

where $\phi(\{i\})$ is the value of bank i with renegotiations formulated in Section 2. The social planning problem is formulated analogously to (20).

Given the lack of explicit closed-form solutions for clearing vectors and bargaining values we characterize the optimal contract choices of banks with several numerical examples for the case where there are three banks. We calculate all relevant expectations with Monte-Carlo simulations. We start though with a simple analytical result for the case where banks maximize profits without renegotiating the OTCD contracts ex post.

Result 4 *If banks maximize profits without renegotiating settlements on interbank claims ex-post, then a pure interbank loan cannot be the optimal hedging contract.*

The intuition for the result is simply that the bank's equity holders do not get any gain unless they can pay off their claims in full. However, with reciprocal pure interbank loans, they can never receive more then the face value of the amount they owe. Therefore, such loans are never optimal.

It is interesting that this result does not carry over to the case where banks renegotiate their interbank settlements. Intuition for this can be got from Lemmas 1 and 2 for the two bank case. There we showed that with renegotiations, bank i is liquidated only when $\tilde{A}_i - L_i + d(\{i\}) < 0$, otherwise the remaining bank could extract resources from bank i without it incurring liquidation costs. Since $d(\{i\}) \geq 0$, this liquidation threshold is lower than for the case without any hedging. Therefore, with renegotiations, pure interbank loans may be optimal.

3.2 Optimal Effort and Contract Choice

In this and the following subsection we provide some numerical results to shed further light on the effort and contract choices of banks. We choose the parameters of our model to approximate the profiles of Baa-rated banks. We fix the parameters μ_0 and μ_1 so that with the endogenously chosen effort, the banks' asset-to-liabilities ratios are about 1.15. The high leverage is consistent with empirical estimates of leverage for banks (see Berger, DeYoung, Flannery, Lee, and Özde Öztekin (2008)). We choose an asset volatility parameter, σ, and a 4-year time horizon for pricing the OTCD contracts. The σ is chosen so that with the above leverage choice we obtain an endogenously determined default probability (PD) or around 1.2 %, which is the average historical 4-year cumulative PD for Baa-rated bonds by Moody's. We first present results with a level of $\mu_1 = 0.5$, a parameter that categorizes the effort incentive for the banks. Later, we consider the implications of a low μ_1.

The OTCDs cause externalities in effort choices through two channels. First, when the bank enters into asset swaps, it cannot capture the full benefit of its investment in effort. It will pass on $(1 - (N-1)\,b)$ of the increase in the mean of $\tilde{A}$ to the other banks, while it still bears the full cost of effort.[14] The optimal effort choice will therefore decrease in b. For interbank loans and CDS contracts this externality is smaller. In the former case the bank's equity holders are the residual claimant of the assets after the interbank loan is paid off, and so they still have a strong incentive to maintain the quality of the assets. This point is illustrated in the bottom left panels of Figure 5, which plots the optimal effort choice for different amounts of a and b contracted. As reasoned, an individual bank's effort decreases in b but is relatively insensitive to a in this example. For CDS contracts, the interbank liability of the bank depends on the performance of *other banks*

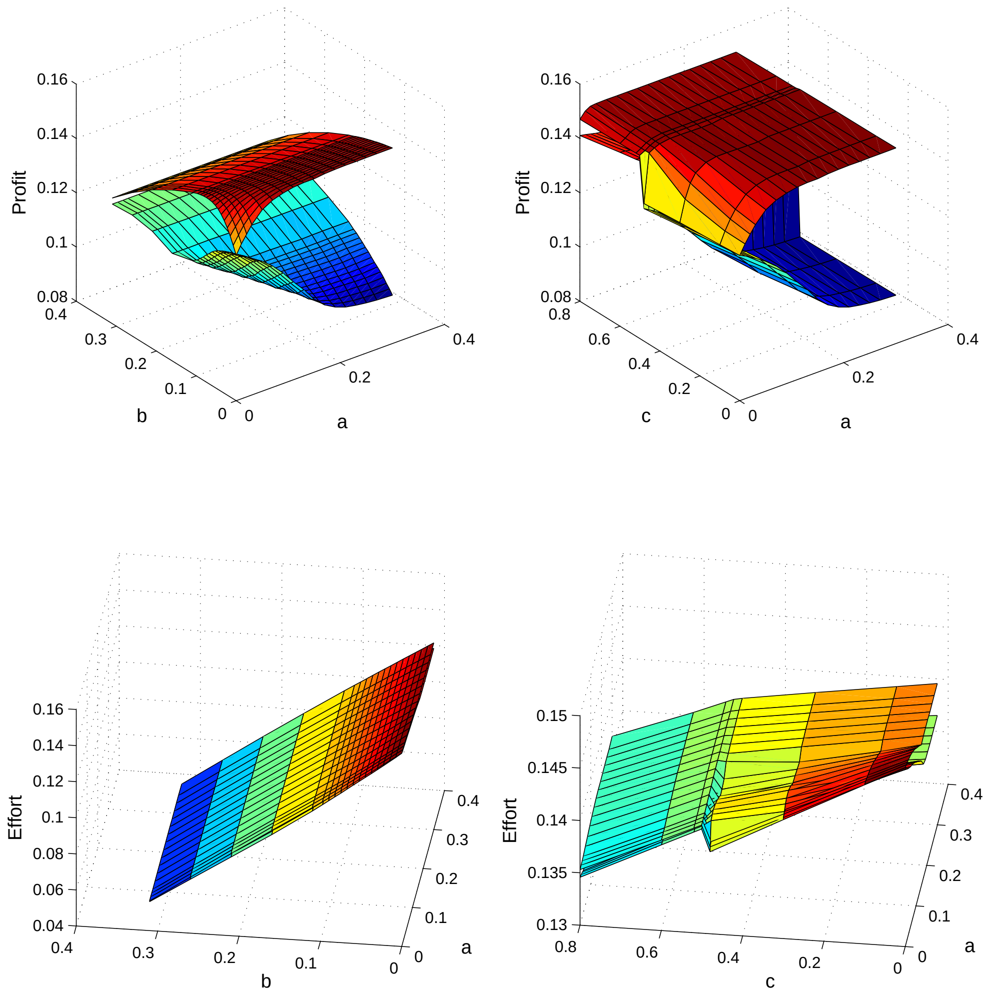
[14] If there are some insolvencies, then this effort mainly benefits the bank's counterparties and helps the bank only through improved recoveries on its claims.

and not the bank's own assets, and hence again the adverse effect on effort incentive is smaller than for asset swaps. The bottom right panel of Figure 5 shows this relative smoothness (notice the smaller scale on the vertical axis) as well as a positive jump in the effort when $c = 0.5$. At $c = 0.5$ each bank *completely* hedges its downside risk with CDS contracts, which boosts recoveries for its counterparties. Therefore, an increase in a bank's effort improves recoveries for the banks' contracts in case of insolvencies of its counterparties and the bank has an incentive to provide it.

The bank's profit for a given risk sharing agreement (a, b, c) is determined by the optimal effort decision given that contract, the benefit from risk sharing (diversification), and the conservation of liquidation costs through renegotiation. The top left panel of Figure 5 illustrates the bank's profit for different a and b choices holding $c = 0$. The upper surface is the profit with renegotiations and the lower surface for the case with the clearing vector. For any given choice of contract values a and b, profits are always higher with renegotiations, because the lowest payoff with renegotiations is in the case where they break down, and then the payoff of banks is as determined by the clearing vector (the payoff without renegotiations). The two surfaces join when banks under two conditions (i) When banks do not hedge (a=0, b=0), and (ii) When there is perfect risk sharing (b=1/3), since in this case the ex post values with the hedges of all banks are the same so that they are either all bankrupt or all solvent and can fulfill their OTCD obligations, so no renegotiations are necessary. As can be seen from the plot, the highest profit with renegotiations is obtained from a pure interbank loan with $a = 0.3$. Thi s happens because with a high amount of interbank loans, the chance of a system run shown in Section 2 by any bank goes to zero as neither solvent bank unilaterally has the ability to bail out the insolvent bank. Thus a high amount of interbank loans help to enforce an efficient liquidation policy.

One of the main results of this paper, in line with Result 4, is that pure interbank loans are are not optimal contracts when there are no renegotiations of OTCD contracts. Even though they do not adversely affect the effort choice as much as asset swaps, straight debt contracts are a poor instrument for risk sharing without renegotiation. Due to their inflexible payments without renegotiations, they lead to a greater incidence of insolvency than the other types of contracts. Asset swaps provide better diversification, because in states in which a bank's asset realization is low, its required payment is low as well, and thus the bank is less likely to be insolvent, and its equity holders can retain positive value. Payments on CDS contracts only occur in periods when the other banks are in trouble, and thus are also less costly then interbank loans in reducing liquidation costs. However, once we allow renegotiations, we find that interbank loans become very useful hedging contracts, at least in the strong bankruptcy regime. In fact, we see from the top right panel of Figure

Figure 5: Bank Profits and Effort Choices for Alternative OTCD Contracts in the Strong Bankruptcy Regime



We display the profit (top panels) and effort choices (bottom panels) of the individual bank for different exposures of straight debt a and asset swaps b. The upper surface is with renegotiations, the lower surface for the clearing vector. The parameter values used for the results are: $\mu_0 = 0.1, \mu_1 = 0.5, \gamma = 2, \rho = 0.1, \sigma = 0.122, \phi = 0.3$.

5 that the banks profits from an optimally chosen large quantity of interbank loans are only slightly higher than that they can achieve with CDS contracts, because effectively the payoff on a renegotiated interbank loan contract is similar to that of a CDS contract (solvent bank makes a payment to an insolvent bank). Notice that this 'replication' happens in our model endogenously due to renegotiations.

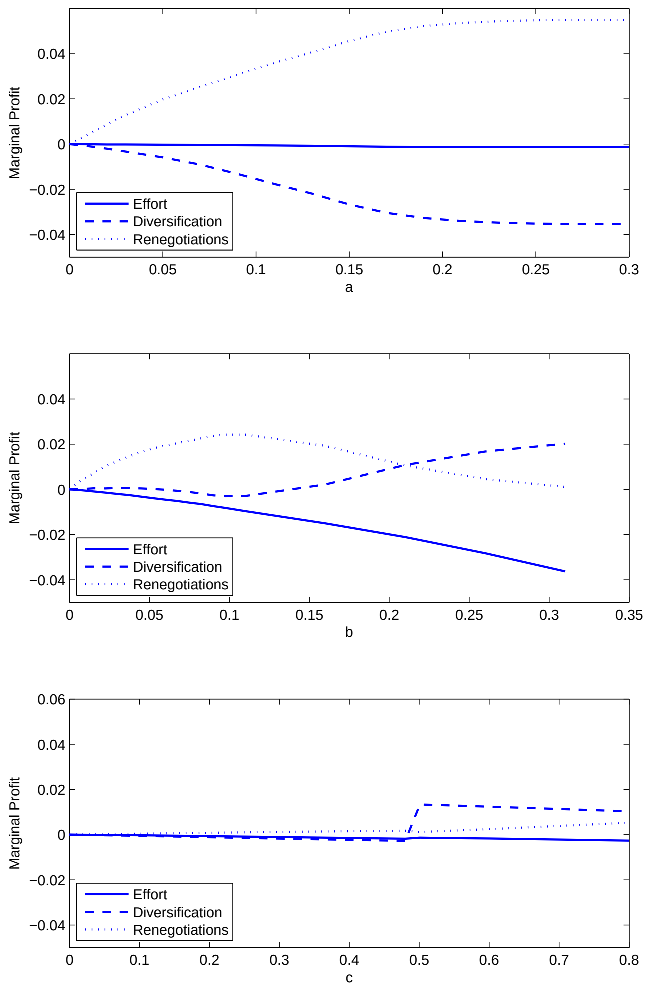
Overall, comparing the performance of the three types of contracts in Table 1 we find that the effort inducement from interbank loans and CDS contracts in the strong bankruptcy regime with renegotiation are very similar. The large optimally chosen quantity of interbank loans helps increase the profits by reducing the small probability of inefficient liquidations that we have with CDS contracts to zero. Recall, that runs occur when an insolvent bank owes amounts to two solvent banks, and the amount owed is not too large so that both solvent banks need to cooperate for the bailout. In contrast, the profit from the swap contract is lower than the other two because of the adverse effect on effort as discussed earlier. The banks therefore remain unhedged and are also subject to a high proportion of runs.

Without renegotiations, the payment streams on the interbank loans do not resemble those of CDS, and the latter dominate in the optimal contract.

To shed further light on the role of the three types of contracts on optimal choices we investigate the marginal impact of each type of contract on banks' profits through three different channels in our model: (i) Impact on effort; (ii) Impact on diversification; and (iii) Impact on Renegotiation and runs. These marginals for each contract are shown in Figure 6. Then in Figure 7 we show the optimal quantities of the three types of contracts if held in isolation. The left panels show banks profits in the strong regime. The top panel of the marginals figure shows that an increase in a has little impact on effort, has a negative impact through diversification, and a positive impact through renegotiations. The top panel of the optimal choices shows that with renegotiations, profits monotonically increase in a. In contrast, with the clearing vector (without renegotiations) profits monotonically decline in a. This clearly shows that interbank loans payouts are very inflexible and cause a lot of financial distress without renegotiation. In particular, higher a implies higher expected liquidation costs that lead to a decline in profits, which is the diversification effect. With renegotiations, this decline is offset by an increase in recoveries as higher a leads to a blocking of inefficient liquidations. When a is high enough, an solvent bank unilaterally cannot perform the bailout, so that the other solvent bank cannot run and leads to the best recoveries from renegotiation.

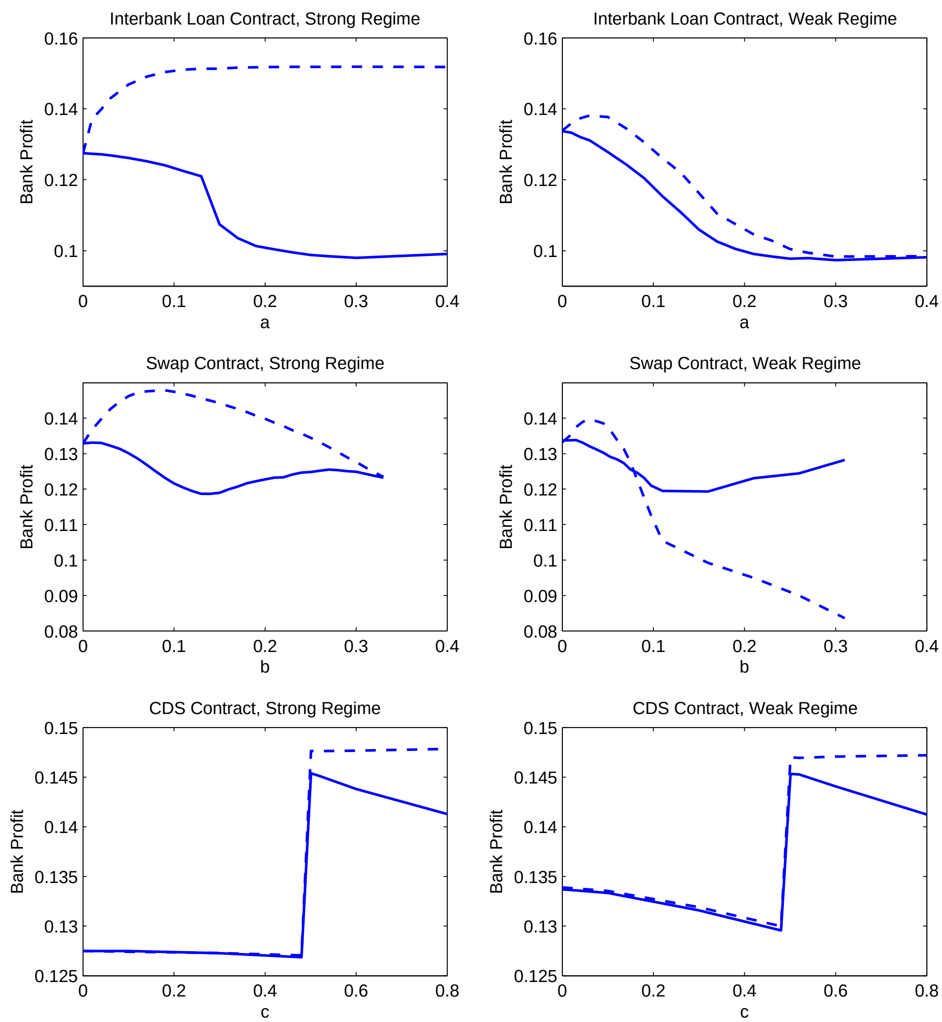
The middle panels of Figure 6 show that asset swaps have the opposite tradeoff. Firstly, with higher swaps, banks exert less effort in boosting the mean of their assets. Having higher b has a

Figure 6: Marginal Profits from Effort, Diversification, and Renegotiation for OTCD Contracts



We display the marginal profits from three different sources for each type of contract. The line labeled "Effort" shows the banks' profits by keeping $a = a^*$, $b = b^*$, $c = c^*$, the optimal contract in the strong bankruptcy regime in Figure 5, but letting effort vary for alternative values of a (top panel, interbank loans), b (middle panel, asset swaps), c (bottom panel, CDS) minus the profit at the optimal contract. The line labeled "Diversification" keeps effort fixed at the optimal contract $(e = e^*(a^*, b^*, c^*)$, then measures the difference between the profits from the clearing vector, π^{CV} under alternative values of a, b, and c and the profits from the clearing vector when $a = 0, b = 0$, and $c = 0$. Finally, the line labeled "Renegotiation" takes the difference in profit from renegotiation π^R when $e = e^*$, and from its level at the optimal contract. The parameter values used for the results are: $\mu_0 = 0.1, \mu_1 = 0.5, \gamma = 2, \rho = 0.1, \sigma = 0.122, \phi = 0.3$.

Figure 7: Optimal Choices of Individual Contracts



We display the optimal choices of the three different OTCDs if banks were to choose a single type of contract: we have interbank loans (top panel), asset swaps (middle panel), and CDS (bottom panel). The parameter values used for the results are: $\mu_0 = 0.1, \mu_1 = 0.5, \gamma = 2, \rho = 0.1, \sigma = 0.122, \phi = 0.3$.

positive impact on diversification, while the impact on renegotiations is hump shaped. Initially as b increases, solvent banks have a greater incentive to bail out the insolvent one as their exposure to the insolvent bank increases so that the renegotiation benefit increases. However, for large b the three banks' net profits become more positively correlated, so that there is no one to perform the bailouts. Thats why with perfect hedging either all the banks remain solvent or they all fail. The positive diversification effect for reducing liquidation costs is intuitive. Overall, in Figure 7, we see that profits are hump shaped in b as renegotiations effect dominates for small b, but eventually the low effort lowers profits.

The bottom panels that study the CDS contracts are the most straightforward. The effect on effort is small (since the payoff of the contract depends on other banks profits), and the renegotiation effect is also small, but increasing in c as banks with a higher exposure in the insolvent bank are more likely to carry out the bailout. The greatest impact is through diversification and there is a jump at $c = 0.5$, when all the downside risk of each bank is removed. This diversification benefit is realized with or without renegotiation and both profits lines in Figure 7 display this jump.

The parameter μ_1 measures how much the asset mean can be improved with by the effort of the bank. When a firm hedges its asset value stream, it has a smaller incentive to maintain its mean, and therefore, this element of the model reduces the incentive of banks to hedge their asset streams. This incentive problem is the larges with asset swaps, as discussed earlier. We verify this logic by keeping all parameters the same as in the above table, but setting the parameter $\mu_1 = 0$. We do not show the detailed results but for the case without renegotiations the optimal contract choice is $b = 0.33$ and no interbank loans or CDS contracts, which implies that all banks are completely diversified with the use of asset swaps. The net payment streams are highly correlated, and thus these contracts help to reduce liquidation costs of the system. For the case with renegotiations, we find that the optimal contract has $a = 0.03$, $b = 0.06$, and $c = 0.8$. This choice has less asset swaps and more CDS. This choice enforces a better liquidation policy than the $b = 0.33$ choice that attempts to minimize the number of liquidations. The CDS contracts actually increase the number of liquidations, but some of these liquidations are optimal (see Result 1).

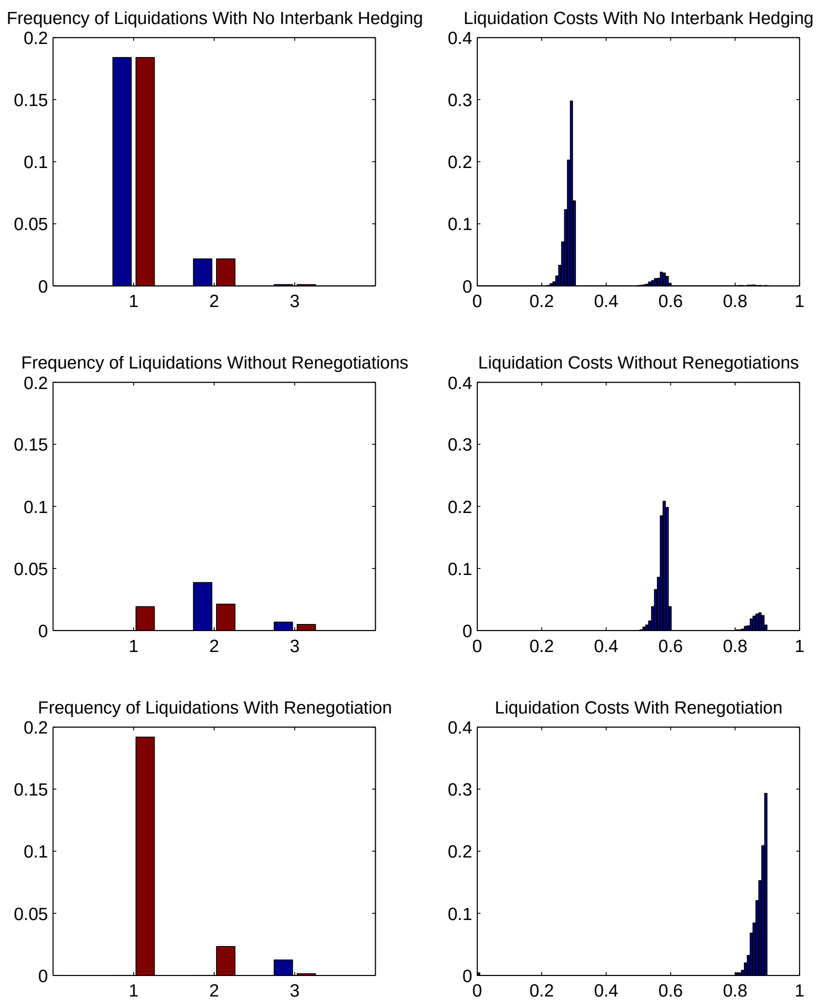
3.3 The Impact of OTCDs on Credit Risk and Systemic Risk in the Strong Bankruptcy Regime.

We now address the main questions of this paper on the effect of OTCDs on credit and systemic risk. Our definition of systemic risk is the frequency of contagious defaults in (5) and (7). We study the simulated frequency of bank liquidations and expected liquidation losses for three cases: with no interbank hedging, with interbank hedging but no renegotiations, and with interbank hedging and renegotiations in Figure 8. We show two sets of bars for each case: the left bar denotes the frequency of liquidations, while the right shows the frequency of technical defaults by banks, some of which are renegotiated and do not lead to liquidations. In comparing the bars though, it is useful to note that some cases of 1 default could be followed by 2 liquidations due to systemic spillovers, and hence the left bar can in principle be higher than the right bar for a given number on the horizontal axis. It is also useful to note that without any interbank hedging the two bars are identical (top panel) as defaults always imply liquidations. It is intuitive that for a given level of effort, hedging and renegotiations should both lower credit risk and systemic risk. However, we study these effects in a setting where due to the change in optimal choices of hedging contracts and effort, the impact on both credit and systemic risk is potentially ambiguous.

For the set of parameters used in the example above, on the one hand renegotiations reduce risk sharing, which increases the variance of the banks' payoffs and thus increases systemic risk. On the other hand, with fewer assets swapped out, banks will optimally increase their effort choice, which raises the mean of the banks payoffs and lowers systemic risk. The benchmark case, with no OTCDs, in the top panels, shows that the probability of at least one liquidation is about 17%, however the probability of 3 defaults is negligible. Moving to the middle panel, the case of hedging with OTCDs and no renegotiations, we find that that the probability of at least 1 liquidation declines to 2%, however the probability of 2 and 3 liquidations increase substantially. All incidences of 3 liquidations are from systemic spillovers and not from fundamental defaults at all three banks simultaneously. Therefore, the use of OTCDs without renegotiations indeed seem to increase systemic risk, although they lower credit risk. Finally moving to the bottom panel, we see that the renegotiations help in removing all instances of 2 liquidations as well, but the chance of 3 liquidations get even stronger. Note that at the optimal contract with renegotiations have a large proportion of interbank loans and hence the chance of 1 technical default rises to 18 percent, however, all such defaults get renegotiated. Overall, credit risk declines further while the incidence of systemic risk (3 liquidations), increases further. It is important to note that the probability of three liquidations from 3 technical default is close to zero, and the proportion of inefficient liquidations in Table 1 is zero, so that all systemic concerns arise from an optimal liquidation policy in which all banks are liquidated simultaneously.

It is also useful to note that bank profits, as exhibited in Figure 5 are higher with renegotiations, so that social welfare (note that depositors losses are covered by deposit insurance in both settings,

Figure 8: Distribution of Default Frequency and Expected Loss with OTCDs in the Strong Bankruptcy Regime



In the **left panels**, we plot two bars at each integer 1,2,3, the possible number of liquidations or fundamental defaults that will be observed in a three bank system. The left and right bars display the frequency of liquidations and fundamental defaults, respectively. We present the simulated frequency for three cases: with no interbank hedging (**top panels**), with interbank hedging but no renegotiations (**middle panels**), and with interbank hedging and renegotiations (**bottom panels**). The **right panels** shows histograms of dead weight losses given at least one default for the three cases. The parameter values used for the results are: $\mu_0 = 0.1, \mu_1 = 0.5, \gamma = 2, \rho = 0.1, \sigma = 0.2, \phi = 0.3$.

whose costs are subtracted from profits) is higher with higher systemic risk. The right panels show that the distribution of expected liquidation losses, which also show an increased positive skewness (correlation in liquidations) in the three cases discussed.

3.4 The Impact of OTCDs on Credit Risk and Systemic Risk in the Weak Bankruptcy Regime.

In this subsection we study the impact of OTCDs on both systemic and credit risks in a weak bankruptcy regime, with and without renegotiations. We recall that the weak regime results from choosing the fixed point of the clearing vector with the smallest payments. As discussed in Example 1, this case, leads to the largest systemic risk.

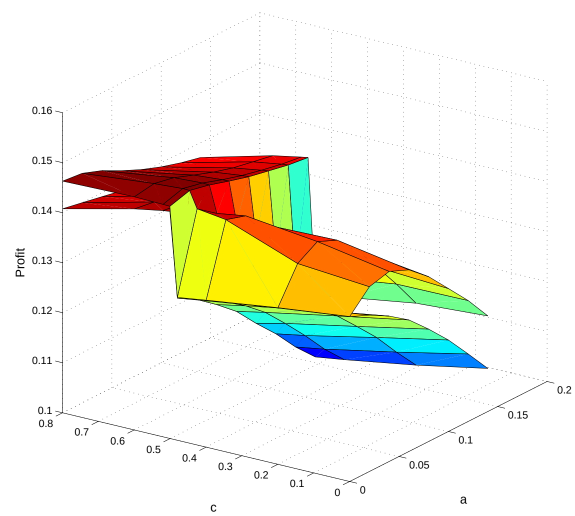
We first consider the optimal OTCD contract choice in this regime when there is a large incentive parameter μ_1. The results for effort are similar to those for the strong bankruptcy case, so are not shown again. In Figure 9, we show the profit plot as a function of the choice of interbank loans and CDS contracts. In contrast to the strong bankruptcy case, we see that the large proportion of CDS contracts are needed to generate optimum profits. The reason why CDS dominates interbank loans is that the clearing vector fixed point gives very low recovery values for interbank loans, which reduce the bargaining values of banks. They therefore avoid their use and instead use CDS. The realized profits are lower then in the strong regime, and there is a positive probability of inefficient liquidations (runs), because these inefficient liquidations could only be driven to zero by choosing very large quantities of interbank loans and thus forcing banks to coordinate bailouts.

The results on systemic risk are in Figure 10, and the panels are analogous to those discussed for the strong regime. The major difference in the two sets of figures is that in the weak regime due to the choice of CDS contracts as opposed to interbank loans, there is a greater incidence of inefficient liquidations due to system runs, and thus not all the technical defaults can be renegotiated away. Overall, somewhat surprisingly the positive skew in liquidation costs is larger as there are more frequent liquidations of one and two banks.

3.5 The Effects of Changing the Bankruptcy Cost Parameter and Asset Correlation

In this section we analyze the comparative static effects of different correlations and bankruptcy costs on bank's profits and the importance of renegotiations. For each pair of correlation and

Figure 9: Bank Profits for Alternative OTCD Contracts in the Weak Bankruptcy Regime

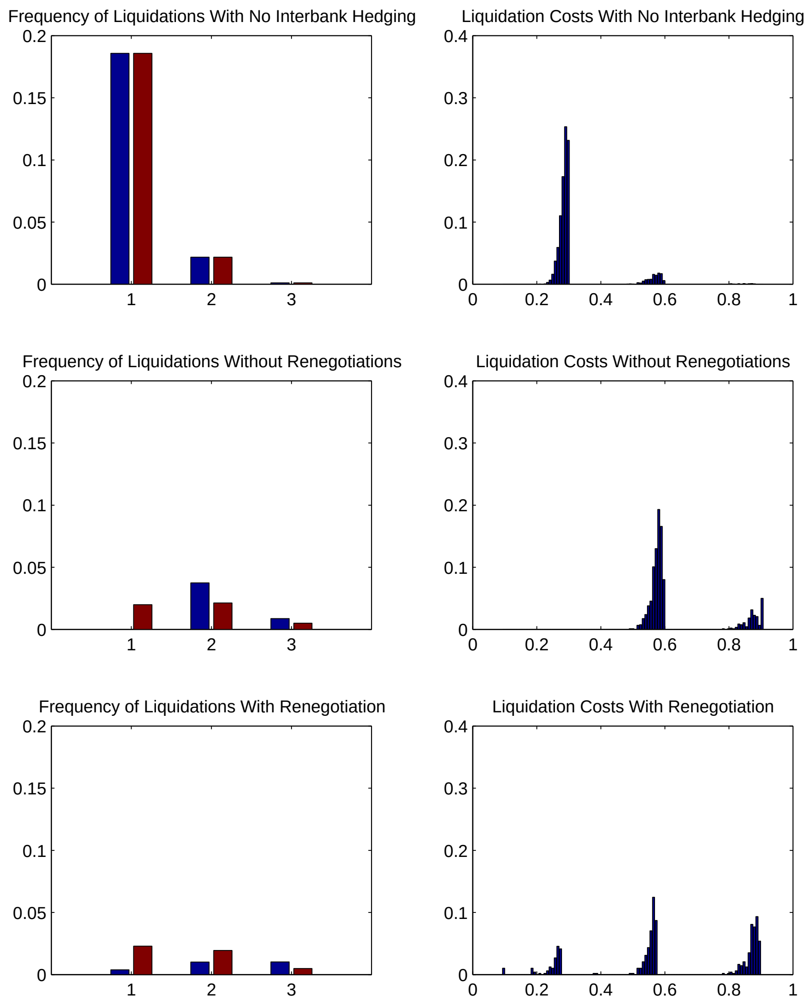


We display the profit of the individual bank for different exposures of straight debt a and CDS contracts c. The upper surface is with renegotiations, the lower surface for the clearing vector. The parameter values used for the results are: $\mu_0 = 0.1, \mu_1 = 0.5, \gamma = 2, \rho = 0.1, \sigma = 0.122, \phi = 0.3$.

bankruptcy we compute the bank's profit for possible contracts (a,b) given an optimal effort choice. We then pick the contract that maximizes the bank's profit and record this profit as the maximum attainable profit. Figure 11 shows the maximum attainable profit for different correlations and bankruptcy costs with and without renegotiated OTCD payments. When bankruptcy costs move to zero, there are no dead weight losses to the system, and hence there is no pie to renegotiate about. Therefore renegotiations become unimportant and profits under the clearing vector and under negotiations converge. We see that a higher Φ on its own hurts bank profitability but renegotiations can mitigate the effect as banks can avoid paying the bankruptcy cost. Therefore the gap between the two surfaces increases for higher values of Φ.

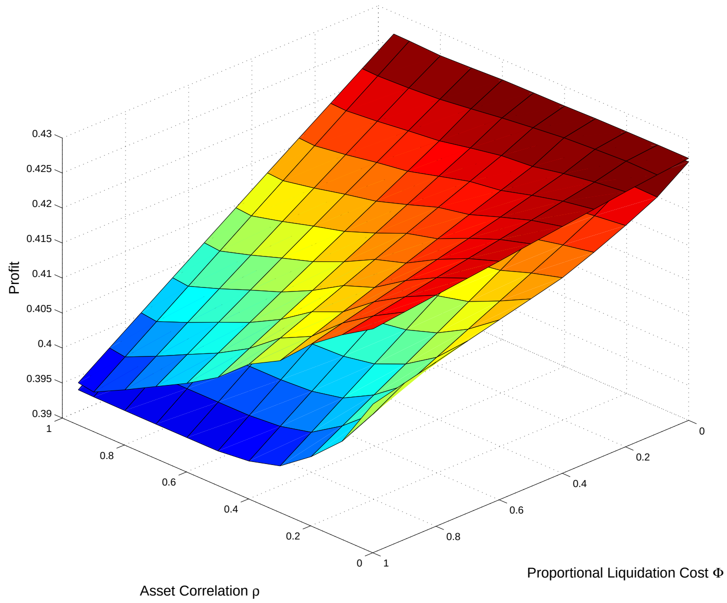
We pursue this last point further by studying the comparative static effect of varying the liquidation cost parameter on total liquidation cost, which is an increasing function of both credit and systemic risks. We study the partial effect of renegotiations by holding the effort and OTCD contract choices fixed. The results in Figure 12 show that liquidation costs increase less rapidly than the parameter itself. This happens, because the proportion of successful renegotiations increases in the liquidation cost parameter. As liquidation costs increase, the threat points of individual banks are lower, so that the chances of renegotiation breakdown decline.

Figure 10: Distribution of Default Frequency and Expected Loss with OTCDs in the Weak Bankruptcy Regime



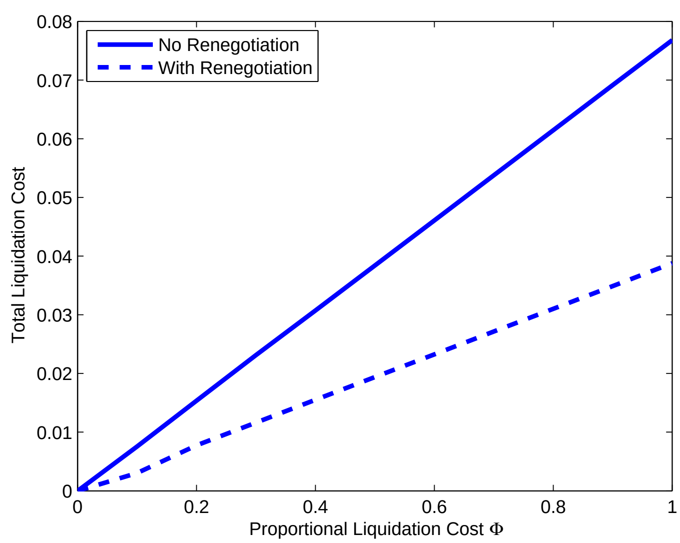
In the **left panels**, we plot two bars at each integer 1,2,3, the possible number of liquidations or fundamental defaults that will be observed in a three bank system. The left and right bars display the frequency of liquidations and fundamental defaults, respectively. We present the simulated frequency for three cases: with no interbank hedging (**top panels**), with interbank hedging but no renegotiations (**middle panels**), and with interbank hedging and renegotiations (**bottom panels**). The **right panels** shows histograms of dead weight losses given at least one default for the three cases. The parameter values used for the results are: $\mu_0 = 0.1, \mu_1 = 0.5, \gamma = 2, \rho = 0.1, \sigma = 0.122, \phi = 0.3$.

Figure 11: Optimal Contract Choice as Functions of Asset Correlation and Proportional Bankruptcy Costs



Profit of the individual bank for different correlations ρ and bankruptcy costs ϕ given the banks optimal contract choice. The parameters are: $\mu_0 = 0.1, \mu_1 = 0.5, \gamma = 2, \rho = 0.1, \sigma = 0.122, \phi = 0.3$.

Figure 12: Liquidation Costs as a Function of the Liquidation Cost Parameter Φ



The parameters are: $\mu_0 = 0.1, \mu_1 = 0.5, \gamma = 2, \rho = 0.1, \sigma = 0.122, \phi = 0.3$.

Returning to Figure 11, we see that increased asset correlation decreases the banks profit at least when bankruptcy costs are positive. A higher correlation decreases the risk sharing opportunities, increases defaults and thus dead weight costs. Renegotiation can again mitigate the problem but become increasingly ineffective as the correlation between the assets approaches 1. In summary, from a set of ex-ante equally profitable investment opportunities, banks will chose ones with low correlation and low liquidation costs.

4 Conclusions and Extensions

We study the role of OTCD contracts in enabling banks to better hedge the risks in their asset streams but in generating greater systemic risk — the risk of financial distress spreading through the financial system — due to the linkages created by these contracts. By swapping out portions of their asset streams, banks lose the incentive to maintain the quality of their assets and compensate for lower quality with greater hedging. Banks attempt to renegotiate their OTCD contracts ex post in the event of insolvencies at one or more banks to lower liquidation costs in the system. Renegotiations helps restore incentives but are unable to improve the equilibrium to the social optimum because they may break down and lead to inefficient liquidations. Breakdowns are endogenous to our model and occur in periods when several solvent banks are able to credibly threaten to 'run' from their obligations to weaker banks. The systemic transmission is greatest in periods of renegotiation breakdown. We show that for banks with low incentive problems assets swaps are the best tools for risk management against system-wide swaps. For banks with high incentive problems, interbank loans and CDS contracts have similar payoffs in strong bankruptcy regimes, but the latter dominate in weak regimes. Optimally chosen OTCDs help increase social welfare and bank profits, lower credit risk, but increase the systemic risk of the system.

While the current version of the paper has investigated the optimal contracts with three banks, in later versions we should be able to extend the analysis to a larger number of banks. It would be interesting to characterize the renegotiation breakdown probabilities as the number of banks becomes large, approximating a perfectly competitive system. We consider our findings a contribution to the recent research agenda set forward by Shiller (2003) to study the optimal design of hedging contracts that could increase the efficiency of the entire banking system, as opposed to individual banks.

Appendix

Proof of Result 1. For establishing the sufficiency of the conditions for a liquidation, if $\tilde{e}_1^{\mathcal{F}} + \tilde{e}_2^{\mathcal{F}} < 0$ then by cooperating and not liquidating either bank the sum of the two banks' equities is negative, while liquidation of either (or both) implies a payoff of $v^{\mathcal{F}}(\{1\}) + v^{\mathcal{F}}(\{2\}) \geq 0$, so that liquidation is an optimal policy. If $\tilde{e}_i^{\mathcal{F}} + \tilde{e}_j^{\mathcal{F}} > 0$, but (13) holds, $\tilde{e}_i^{\mathcal{F}} < 0$, and by (12) $p^{\mathcal{F}}(\{i\}) = 0$. Therefore we have $v^{\mathcal{F}}(\{i\}) = 0$ and (14) implies that $v^{\mathcal{F}}(\{j\}) = \tilde{e}_j^{\mathcal{F}} - d^{\mathcal{F}}(\{j\}) > 0$. Under (13), $\tilde{e}_j^{\mathcal{F}} - d^{\mathcal{F}}(\{j\}) > \tilde{e}_i^{\mathcal{F}} + \tilde{e}_j^{\mathcal{F}}$ so that liquidation of i gives the two players a larger amount to share relative to cooperating and canceling out all OTCD contracts.

For establishing the necessity of the stated conditions suppose (13) does not hold while (14) does. Then we have $\tilde{e}_i^{\mathcal{F}} + d^{\mathcal{F}}(\{j\}) > 0$ and $\tilde{e}_j^{\mathcal{F}} - d^{\mathcal{F}}(\{j\}) > 0$. Clearly bank j can meet its OTCD commitments in full. There are two subcases: (i) $0 \leq p^{\mathcal{F}}(\{i\}) < d^{\mathcal{F}}(\{i\})$, then $v^{\mathcal{F}}(\{i\}) = 0$, and and $v^{\mathcal{F}}(\{j\}) = \tilde{e}_j^{\mathcal{F}} - d^{\mathcal{F}}(\{j\}) + p^{\mathcal{F}}(\{i\}) < \tilde{e}_j^{\mathcal{F}} - d^{\mathcal{F}}(\{j\}) + \tilde{e}_i^{\mathcal{F}} + d^{\mathcal{F}}(\{j\}) = \tilde{e}_1^{\mathcal{F}} + \tilde{e}_2^{\mathcal{F}}$, where the last inequality obtains because (12) implies that $p^{\mathcal{F}}(\{i\}) < \tilde{e}_i^{\mathcal{F}} + d^{\mathcal{F}}(\{j\})$. (ii) If $p^{\mathcal{F}}(\{i\}) = d^{\mathcal{F}}(\{i\})$ then both banks are able to pay in full, and hence $v^{\mathcal{F}}(\{1\}) + v^{\mathcal{F}}(\{2\}) = \tilde{e}_1^{\mathcal{F}} + \tilde{e}_2^{\mathcal{F}}$. Thus cooperation is the optimal strategy for each case.

Next (13) holding and (14) not holding contradicts $\tilde{e}_i^{\mathcal{F}} + \tilde{e}_j^{\mathcal{F}} > 0$. To complete the proof of the sufficiency of the conditions we need to consider the cases when $\tilde{e}_i^{\mathcal{F}} + \tilde{e}_j^{\mathcal{F}} > 0$ and neither condition holds. We then have $\tilde{e}_i^{\mathcal{F}} + d^{\mathcal{F}}(\{j\}) > 0$ and $\tilde{e}_j^{\mathcal{F}} - d^{\mathcal{F}}(\{j\}) < 0$. Again breaking up the analysis into cases we have: (i) If $0 \leq p^{\mathcal{F}}(\{i\}) < d^{\mathcal{F}}(\{i\})$, then $v^{\mathcal{F}}(\{i\}) = 0$. Then $v^{\mathcal{F}}(\{j\}) \leq \tilde{e}_j^{\mathcal{F}} - d^{\mathcal{F}}(\{j\}) + p^{\mathcal{F}}(\{i\}) < \tilde{e}_j^{\mathcal{F}} - d^{\mathcal{F}}(\{j\}) + \tilde{e}_i^{\mathcal{F}} + d^{\mathcal{F}}(\{j\}) = \tilde{e}_1^{\mathcal{F}} + \tilde{e}_2^{\mathcal{F}}$ as in case (i) of the previous paragraph. Note that the first inequality is strict unless $\tilde{e}_j^{\mathcal{F}} - d^{\mathcal{F}}(\{j\}) + p^{\mathcal{F}}(\{i\}) > 0$. Lastly consider the case that bank i can meet its OTCD commitments in full. Then $v^{\mathcal{F}}(\{i\}) = \tilde{e}_i^{\mathcal{F}} + p^{\mathcal{F}}(\{j\}) - d^{\mathcal{F}}(\{i\})$, and $v^{\mathcal{F}}(\{j\}) \leq \tilde{e}_j^{\mathcal{F}} - p^{\mathcal{F}}(\{j\}) + d^{\mathcal{F}}(\{i\})$, with equality holding only if bank j can pay its OTCDs in full. Summing the two we have $v^{\mathcal{F}}(\{i\}) + v^{\mathcal{F}}(\{j\}) < \tilde{e}_1^{\mathcal{F}} + \tilde{e}_2^{\mathcal{F}}$ so that liquidation is not optimal. ■

Proof of Result 2. In case (i) both banks are able to meet their OTCD commitments. Then irrespective of the bidding order neither bank will accept a bid less than $v^{\mathcal{F}}(\{i\})$. The sum of these reservation values in this case is simply $\tilde{e}_1^{\mathcal{F}} + \tilde{e}_2^{\mathcal{F}}$, the value to be shared by merging the banks. Therefore, there are no net gains from merging irrespective of the order of proposals.

Next consider case (ii): equations (13) and (14) hold as well as $\tilde{e}_1^{\mathcal{F}} + \tilde{e}_2^{\mathcal{F}} > 0$, so that it is efficient to liquidate bank i. Suppose that bank j is the first mover and can make a take-it-or-leave-it offer to assume both the assets and liabilities of bank i as well as all its OTCD commitments. Since $v^{\mathcal{F}}(\{i\}) = 0$, bank j will never make a positive bid. However, under the stated conditions, even a zero bid by bank j is not optimal, since by purchasing bank i for zero it shareholders will obtain a combined value of $\tilde{e}_1^{\mathcal{F}} + \tilde{e}_2^{\mathcal{F}} < \tilde{e}_j^{\mathcal{F}} - d^{\mathcal{F}}(\{j\})$, the value it gets if bank i is liquidated. However, bank i will not accept a negative bid because its equity holders can obtain $v^{\mathcal{F}}(\{i\}) = 0$ by liquidating. Consider now the case where bank i gets to bid first. The lowest bid that bank j will accept is $v^{\mathcal{F}}(\{j\}) = \tilde{e}_j^{\mathcal{F}} - d^{\mathcal{F}}(\{j\})$ and by making such a bid and merging the two banks, the shareholders of bank i will have a combined value of $\tilde{e}_1^{\mathcal{F}} + \tilde{e}_2^{\mathcal{F}} - \tilde{e}_j^{\mathcal{F}} - d^{\mathcal{F}}(\{j\}) < 0$. Therefore, irrespective of the bidding order no successful bid can be made to merge the banks and bank i will be liquidated.

The proofs of cases (iii) and (iv) are similar and are omitted for brevity. ■

Proof of Result 3.

(i) We solve the game by backward induction. Start with node E. At this point, banks 1 and 2 are independent and bank 3 has rejected bank 1's bid. Thus the only player that can potentially sign up 3 is bank 2. Bank 2 bids b_2^E for bank 3 in node E. When signing up 3, bank 2 collects a payoff of $v^{\{1\},\{2,3\}}(\{2,3\}) - b_2^E$. Bank 2 is willing to bid up to the point where it is indifferent between signing up 3 and staying independent, which is given in (15).

Bank 3 can always realize a payoff of $v^{\mathcal{N}}(\{3\})$ by rejecting bank 2's bid. Thus bank 3 will accept every bid above $v^{\mathcal{N}}(\{3\})$. Since bank 2's payoff is decreasing in b_2^E, it will never bid more than bank's 3 reservation price. Thus in equilibrium we know that

$$b_2^E = \min(\bar{b}_2^E, v^{\mathcal{N}}(\{3\})) \tag{22}$$

The payoffs ϕ^E of the players conditional on reaching node E as well as the realized partition ψ^E are as in the statement of the result.

(ii) The case where both 1 and 2 potentially compete for 3 is more complex. In order for the game to proceed to node F, $b_1^C > \phi^E(\{3\})$ or player 1's bid will be rejected by player 3. Suppose that 1 has made a b_1^C that is not rejected the game moves to node F. Bank 2 knows that partition $\{\{1,3\},\{2\}\}$ forms when it loses the bidding war with 1. A similar logic to the case above defines the maximum $\bar{b}_2^F$ that bank 2 given in (17). Bank 2 can win the bidding war if it bids just above b_1^C, but bank 2 would never bid more than $\bar{b}_2^F$. Thus bank 2's bid is $b_2^F = \min(b_1^C, \bar{b}_2^F)$. Similarly, bank 1 will bid satisfies $b_1^C = \min(\bar{b}_1^C, b_2^F)$, where $\bar{b}_1^C$ is given by (16). The first term shows the gain of bank 1 winning the bidding war with 2 rather than losing, while the second term ensures that the game proceeds to node F. Now, if $\bar{b}_2^F > \bar{b}_1^C$ then 2 wins the bid as stated.

(iii) Follows from bank 1's optimizing decision at node C, anticipating the payoffs at nodes E and F.

(iv) Now consider the subgame at node D. Bank 2 has signed up with 1 for a bid of b_1^B and now player 1 thinks about signing up bank 3 as well. Similar to the case above, bank 1's maximum bid for bank 3 is $\bar{b}_1^D$, which makes it indifferent between merging with bank 3 or being independent, is given by (18). There is no competition for bank 3 and its reservation price is what bank 3 can get by itself which is $v^{\{1,2\},\{3\}}(\{3\})$. Bank 1 would therefore never bid more than that and bank 1's equilibrium bid is

$$b_1^D = \min(\bar{b}_1^D, v^{\{1,2\},\{3\}}(\{3\})). \tag{23}$$

(v) We can now determine the final outcome of the game by examining node B. Bank 1 has to decide whether or not to sign up bank 2. It compares the payoff of moving to node D and collecting a payoff of $\phi_1^D(b_1^B)$ with moving to node C and collecting payoff of ϕ_1^C. It is willing to bid for 2 in node B up to the point of indifference where $\phi_1^D(\bar{b}_1^B) = \phi_1^C$, or

$$\max(v^{\{1,2\},\{3\}}(\{1,2\}) - \bar{b}_1^B, v^{\{1,2,3\}}(\{1,2,3\}) - v^{\{1,2\},\{3\}}(\{3\})) - \bar{b}_1^B) = \phi_1^C.$$

Bank 2's reservation price is what it can get by not merging with 1, thus it will accept all offers above ϕ_2^C. Bank 1 will therefore bid in equilibrium $b_1^B = \min(\bar{b}_1^B, \phi_2^C)$.

This completes the analysis of the extensive form game. ■

Proof of Result 4. To facilitate the exposition of the proof, we reverse the order of the choice of the contract terms and effort in (19), and write the profit of bank i as

$$\pi^{CV}(\{i\}) = \text{Max}_{h_i}\left[\text{Max}_{a(h_i)}\left[E\left[v^{\mathcal{N}}(\{i\}) - \omega^{D,CV}(\{i\}) - \gamma \cdot h_i^2\right]\right]\right],$$

that is we first fix the effort level of the bank, and then choose the contract terms as a function of this effort level. Take any effort level h^*, fix $b = 0$ as presumed and consider the optimal choice a. With the effort choice fixed, the distributions of $\tilde{A}_i$, $i = 1, 2, 3$ are fixed. Now with a choice of any $a > 0$, the ex-post profit conditional on any realization of the asset values from the clearing vector for i is $\max(\tilde{A}_i - L_i + r^{\mathcal{N}}(\{i\}) - (N-1)\, a, 0)$, while with $a = 0$, the profit would be $\max(\tilde{A}_i - L_i, 0)$. However, by the definition of the clearing vector in (2) we have that $r^{\mathcal{N}}(\{i\}) \leq (N-1)a$, which implies that the profit with $a = 0$ is greater than or equal to the profit with $a > 0$. This completes the proof. ■

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Table 1: Bank Effort Choice, Profits and Systemic Risk Measures for Alternative CDS Contracts

	a	b	c	Effort	Profit	PD	Inefficient Liquidations	PD Fund.	PD Contagious
					Strong Bankruptcy Regime				
With Renegotiations									
Optimal Contract	0.30	0	0	0.140	0.152	1.20%	0	0.75%	0.45%
Interbank Loans	0.30			0.140	0.152	1.20%	0	0.75%	0.45%
Swaps		0.08		0.123	0.128	2.01%	45.85%	1.46%	0.55%
CDS			0.50	0.140	0.148	2.82%	0.24%	2.31%	0.51%
No Renegotiations									
Optimal Contract	0	0	0.50	0.140	0.145	3.36%	14.43%	2.39%	0.97%
Interbank Loans	0			0.147	0.134	7.68%	0	0.00%	7.68%
Swaps		0		0.147	0.134	7.68%	0	0.00%	7.68%
CDS			0.50	0.140	0.145	3.36%	14.43%	2.39%	0.97%
No hedging	0	0	0	0.147	0.133	7.75%	0	7.75%	0.00%
					Weak Bankruptcy Regime				
With Renegotiations									
Optimal Contract	0.02	0	0.52	0.139	0.148	1.79%	4.56%	1.39%	0.40%
Interbank Loans	0.04			0.146	0.138	3.56%	52.66%	3.35%	0.21%
Swaps		0.05		0.131	0.137	2.54%	54.82%	2.26%	0.28%
CDS			0.60	0.138	0.124	2.95%	9.10%	2.04%	0.91%
No Renegotiations									
Optimal Contract	0	0	0.50	0.140	0.145	3.37%	14.63%	2.25%	1.12%
Interbank Loans	0			0.147	0.133	7.75%	0	7.75%	0.00%
Swaps		0		0.147	0.133	7.75%	0	7.75%	0.00%
CDS			0.50	0.140	0.145	3.37%	14.63%	2.25%	1.12%

Results are obtained obtained from Monte-Carlo simulation of banks asset value processes as in Assumption 1 and the renegotiation process in Section 2. The parameter values used for the simulation are: $\mu_0 = 0.1, \mu_1 = 0.5, \gamma = 2, \rho = 0.1, \sigma = 0.122, \phi = 0.3$. PD stands for probability of default. Inefficient Liquidations is the proportion of inefficient liquidations relative to total liquidations. A liquidation policy is inefficient if $w^{\mathcal{F}}(S) > v^{\mathcal{F}}(S)$, where w is the value of the collection of banks S with an optimal liquidation policy and v is the function that is realized from the bargaining process (see (11). PD Fund is the fraction of defaulted banks that are in fundamental default as in (4). PD Contagious is the fraction of defaulted banks that are in contagious default as in (7). The results for the case without hedging do not depend on the bankruptcy regime because there are no linkages between banks.