Government Bailout Policy:

Transparency vs. Constructive Ambiguity

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Abstract

Increasingly, governments are seen to offer capital support to major financial institutions and corporations when they become financially distressed. While such support could avoid economy-wide systemic damages, it also contributes to the well-known problems of moral hazard. This paper discusses optimal government bailout policy where the costs of systemic failures and moral hazard problems are considered. We find that the optimal bailout policy takes different forms: it calls for a guaranteed bailout for big institutions (“Too Big to Fail”), a randomized bailout for medium-sized institutions (“Constructive Ambiguity”), and no bailout for smaller ones(“Too Small to Save”). Under the policy, a transparent, guaranteed bailout of those institutions deemed to be too big to fail, needs to be accompanied by ex post monitoring (auditing) and a penalty scheme in order to deter moral hazard behavior. However, in a volatile, innovative market environment where individual institutions may know more than the government regulator, monitoring error could contribute to risk-taking, leaving the government regulator to always play a “catch-up” role in revising policy. The optimal bailout policy may not be time-consistent: institutions not deemed “too big to fail” may still take excessive risks and expect to be bailed out in case of insolvency. The time inconsistency problem mainly comes from the short-term orientation of the government. Finally, because an institution’s systemic risk affects the probability of a bailout, the boundary of a firm may be extended by the government subsidy.

1 Preliminary. Please do not quote without asking for the authors’ permission. Corresponding author: Ning Gong, Melbourne Business School, 200 Leicester Street, Carlton, VIC 3053, Australia. E-mail: n.gong@mbs.edu. We would like to thank Doug Foster, Ed Kane, Tano Santos, Neil Stoughton, and participants at the Financial Integrity Research Network Research Day presentations in Brisbane and an internal seminar at FDIC for helpful comments.
1: Introduction

As illustrated during the recent financial market turmoil, it is becoming increasingly common for a government authority to intervene in the marketplace to prevent the failure of a systemically-important firm or industry. In 2008, for example, the U.S. government prevented the individual failures of the investment bank, Bear Sterns, the government-sponsored enterprises, Fannie Mae and Freddie Mac, the insurance giant, American International Group, and the nation’s fourth largest commercial bank, Wachovia Bank. Then, the U.S. government took the unprecedented action of implementing a $700 billion bailout of the U.S. financial system. Elsewhere around the world, governments took similar actions. In the United Kingdom, authorities nationalized, at least in part, most of the nation’s banks, including the mortgage banks, Northern Rock and Bradford & Bingley. And in Europe, governments took similar carte blanche actions to rescue financial firms, pledging hundreds of billions of euros to bailout the likes of Fortis Bank (Belgium, Netherlands, Luxenbourg), Sachsen LB (Germany), IKB (Germany), Dexia (Belgium, France, Luxenbourg), Glitner (Iceland), and Hypo Real Estate Holding AG (Germany). While the high number of rescues in 2008 is unusual, world governments have a long history of bailing out economically and politically important firms and industries.²

In this paper, we analyze the incentives created by a government bailout policy. Although our motivation and focus are mainly on the capital support of financial institutions, the results are equally applicable to government rescue policies for non-financial corporations, and may shed light on issues relating to other types of government aid packages, such as emergency aid for farmers and residents harmed by natural disasters. Regardless of industry, the common theme is that the insolvency of a firm has significant negative externality to the economy and/or the incumbent government. For simplicity, we will refer to the recipients of government bailouts as banks in the analysis.

² See, for example, the collection of papers in Gup (2004).
In the current debate on government bailout policy, there are two key questions: (1) Under the doctrine of *laissez-faire*, should the government intervene to save failing institutions? (2) If government bailouts are deemed necessary, then what is the best policy for the government to follow? In other words, how should a bailout policy be structured in order to minimize public costs and to correct for problems associated with moral hazard?

Assuming that the government’s objective is to minimize the total expected costs to taxpayers and society, we have derived the *ex ante* optimal bailout policy which takes different forms based on the systemic cost of an institution’s failure. In short, our three-tiered bailout policy calls for a guaranteed bailout for systemically important (a.k.a “too big to fail”) institutions, but with a penalty attached; a randomized bailout policy for medium-sized institutions (otherwise known as a policy of “constructive ambiguity”); and a “no bailout policy” for smaller banks (“too small to save”).

However, the optimal bailout policy has its drawbacks in practice when imperfections are considered. The transparent rescue policy for those institutions with the largest systemic cost requires that the government be able to make a correct post-event judgment about whether the institution has indeed acted prudently. Asymmetric information about the magnitude of any *ex post* monitoring (auditing) error is a major concern. Significant information asymmetries could make the transparent policy ineffective. The policy of constructive ambiguity may also be ineffective if the policy lacks credibility. If medium-sized banks (those not judged by the government to be “too big to fail”) believe that the government is short term oriented and more likely to resort to a bailout than officially acknowledged, then medium-sized institutions may come to regard themselves as “too big to fail” and engage in moral hazard behavior. This highlights the importance of consistency in the government’s policy.

Our results also explain why smaller institutions may have incentives to merge or expand, in order to take advantage of the government bailout policy and lower their cost of capital. This is an example of how the boundary of a firm may be extended by government policy and how a firm’s systemic impact can be a choice variable. This result can also affect an institution’s risk management practices as well.
1.1 Related Literature

Our paper relates to the literature on bank monitoring, regulation on the one hand, and the dynamic consistency of government policy, on the other. The underlying issue is the moral hazard problem of a principal agent relation between the government regulator and firms.

There is a rich tradition of literature on liquidity crises in the banking sector. Diamond and Dybvig (1983) and Bryant (1980) are among the first to formally model the bank liquidity crises and address the need for a government sponsored deposit insurance scheme to eliminate bank runs. Government support for the banking sector goes beyond preventing liquidity crises and providing liquidity as a lender of the last resort. Sometimes, it is difficult to distinguish liquidity support from capital support when a bank is insolvent. Freixas et al. (2004) analyze the regulator’s role in such an environment, without the consideration of the systemic risk.

In this paper, we will address the issue of government capital support policy for failed large institutions. Although the concept is not restricted to the banking sector, “too big to fail”, when applied to banking, has come to refer to the practice followed by government regulators of protecting uninsured creditors and depositors of large banks from loss in the event of failure. Despite the numerous discussions on the policy of the “too big to fail” doctrine\(^3\), formal modeling of the government bailout policy is lacking, with the exception of Freixas (1999), Goodhart and Huang (1999), and Diamond (2001). In Goodhart and Huang’s (1999) model, the government regulator decides whether to rescue a bank after evaluating the expected cost of rescuing and the desired monetary policy goal, defined as the total amount of deposits in the economy. Diamond (2001) argues that a distressed bank should be recapitalized with public funds only if it protects the value of existing relationship lending and human capital in banks and firms. Surprisingly, Diamond and Rajan (2002) argue that if the aggregate liquidity is limited in the economy, government bailout of illiquid banks may tip a banking system into a

\(^3\) The origins and evolution of the TBTF doctrine in the U.S. are discussed in Holland (1998), Golembe (1991), and Hetzel (1991), among others. For a comprehensive discussion and institutional details of the debate on “too big to fail” doctrine, see Stern and Feldman (2004).
systemic crisis, as more-liquid banks will eventually have to fail as real interest rates get bid up.

Our paper is related to that of Freixas (1999), which discusses the government bailout policy for the uninsured debt-holders of banks. However, there are several differences between the two. First, his model is bank-specific and the decision whether to liquidate or rescue a bank depends on its liability structure. By contrast, our model is simpler and more general. It is not limited to the government rescuing policy for a certain class of claimholders. Second, we discuss the dynamic implications of the pre-committed government bailout policy and address the importance of reputation and penalty in a repeated game setting. The discussion in this regard follows Friedman (1971). A recent, comprehensive treatment of the reputations in repeated games can be found in Mailath and Samuelson (2006). Third, the meaning of the “bailout” is different. We treat “bailout” as open assistance: giving the money to the bank/firm and let them survive. Freixas’ main interpretation is “purchase and assumption,” and therefore the real option value embedded in the continuation of current bank’s operations is considered.

The rest of the paper is organized as follows. After describing the basic set-up of the model in Section 2, we derive step-by-step the ex ante optimal government bailout policy in Section 3 without monitoring and penalty. In Section 4, we discuss the role of monitoring and penalty schemes attached to the bailout policy with respect to large institutions. In Section 5, we investigate the dynamic implications of the ex ante optimal bailout policy for medium-sized institutions which may be encouraged to take excessive risks if they believe that the government is short-term oriented. The endogeneity of a firm’s systemic cost and the effect on the firm’s boundary under the bailout policy are discussed in Section 6. Section 7 concludes.

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4 See the footnote 4 of Freixas (1999), where our case is mostly close to case 2 (although there may be some sort of control exchanged), while his is close to case 3.
2: Model Set-up

In the economy, there are two types of risk neutral agents: banks and a government regulator. The objective function of a bank is to maximize its expected net payoff. The objective function of the government regulator is to minimize the sum of the expected systemic cost and the expected bailout payment in the case of a bank failure.

Banks (firms) can choose between a safer loan portfolio (project) and a riskier, potentially more profitable loan portfolio (project), which will be defined shortly. There is a continuum of bank types, distinguished by the systemic cost of a bank’s failure. As mentioned previously, this systemic cost includes direct bailout costs, direct and indirect market spillover effects, and society’s opportunity cost of providing the bailout. Note, however, that the market impact cost may or may not be proportional to a bank’s asset size. A financial institution with a smaller asset size but very convoluted relationships with other banks may have a significant systemic cost when it fails. For convenience, we sometime refer to these banks with a higher systemic cost as “big” banks.

As more bailouts and subsidies extended to the manufacturing sectors in many developed countries as well, some critics asked whether unemployment caused by large scale layoffs could be considered systemic risk. Putting aside these debates, let us first assume that the market impact cost, \( C \), is exogenously determined, and is known to the bank and the regulator. \( C \) is distributed uniformly on \([0, \bar{C}]\). The timeline of the events is in Figure 1.

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5 Possible spillover effects include depositor runs, domino-like failures of correspondent banks, serious disruptions in domestic and international payment and settlement systems, short-term credit availability problems, and an impairment of public confidence in the broader financial system.

6 The assumption of the uniform distribution is for the purpose of simplicity. Any alternative specification of the probability distribution works fine.
To highlight the incentive issues, each bank has two mutually exclusive potential projects. Without loss of generality, each project is assumed to require an investment of $1. In the case of failure, the bank loses all of its initial investment. The net payoff of project $i$ ($i = 1, 2$) has the distribution:

\[ X_i = \begin{cases} 
R_i & "High" \text{ state with probability of } p_i \\
0 & "Medium" \text{ state with probability of } p_m \\
-1 & "Low" \text{ state with probability of } (1 - p_1 - p_m) 
\end{cases} \]

We assume that $p_1 > p_2$, and $R_2 > R_1 > 0$.

Without any government rescue or subsidy, the expected net payoff for Project 1 is $E(X_1) = p_1 R_1 - (1 - p_1 - p_m)$. Similarly, the expected net payoff for Project 2 is $E(X_2) = p_2 R_2 - (1 - p_2 - p_m)$. Let us assume that, inherently, Project 1 is a better project because it has a higher net expected payoff and a lower risk than Project 2. That is,

\[ p_1 R_1 + (p_1 - p_2) > p_2 R_2 \]  \hspace{1cm} (1)
Obviously, the bank will take Project 1 if there is no subsidy offered by the government.

However, if the government believes that there is a systemic risk involved should the bank become insolvent\(^7\) (in this setting, the payoff of the project reaches the low state and the bank loses all the initial investment), then it will subsidize the bank to bring the net payoff back to zero in the “low” state without any penalty. The revised expected net payoff for the bank with the guaranteed subsidy is \(p_1R_1\) for Project 1, and \(p_2R_2\) for Project 2, respectively. We assume that
\[
p_2R_2 > p_1R_1 \quad (2)
\]
Thus, the bank will choose investment project 2. We show in the Appendix that project 2 is riskier because its expected net payoff has a higher volatility than project 1.

**Lemma 1**: Under the model set-up, project 2 is riskier than project 1.

In this case, the expected amount of subsidy paid by the government is \((1 - p_m - p_2)\) if the second project is chosen, which is higher than the expected subsidy of \((1 - p_m - p_1)\) if the first project is chosen. This clearly illustrates the well-known moral hazard problem when the government offers an unconditional guarantee. While it is not essential to the analysis, it is worth noting that under the subsidy, a negative Net Present Value (NPV) project could be chosen by the bank.\(^8\)

Combining the restrictions imposed on projects (1) and (2),
\[
p_1 R_1 + (p_1 - p_2) > p_2R_2 > p_1 R_1 \quad (3)
\]
Rewriting (3) in the following form will be helpful in our analysis,
\[
0 < p_2R_2 - p_1 R_1 < p_1 - p_2 \quad (4)
\]

\(^7\) Here we are a bit loose in defining insolvency, because we have not introduced the debt in the bank’s capital structure. Also, in practice, when the government bails out a firm or a bank, it may very well take an equity position and dilute the existing shareholders’ interests. The price the government offers is higher than the prevailing market price. Shareholders will lose some money as well, and it is not a complete free-ride for them. As we will see, the dilution of the shareholders’ equity can be part of the penalty associated with the government bailout.

\(^8\) It is easy to verify this by choosing the following parameters: \(p_1 = 0.75, p_2 = 0.4, p_m = 0.15, R_1 = 0.5, R_2 = 1\).
We are now ready to analyze the incentives for project selection by banks under two alternative government rescue policies: transparent vs. ambiguous, and to derive the optimal government bailout policy.

3: Optimal Bailout Policy without Monitoring

We consider a one-shot game in this and the next sections, the dynamic implications of which are explored in Section 5. As depicted in the timeline in Figure 1, the government regulator moves first to set the regulation and the bailout policy in the case of bank failure. Banks then choose investment projects (loan portfolios) accordingly. In the first step, the strategies available to the government are either full subsidy with a probability of 100% when a bank fails; or no subsidy at all. There is no uncertainty with respect to the bailout. A randomized bailout strategy will be introduced later. In doing the analysis incrementally, we can explain the marginal benefit of the inclusion of a randomized bailout policy.

Assume that the government offers no bailout. A bank chooses Project 1 (“good” one). The total expected cost to the government when Project 1 is chosen without subsidy is \((1 - p_1 - p_m)C\). This is due to the fact that the bank will fail with a probability of \(1 - p_1 - p_m\). When the bank fails, the systemic cost is \(C\).

If the bank knows for sure it will always be bailed out in the case of a bad outcome, then it chooses Project 2 (“bad” one). The total expected cost to the government with subsidy is \(1 - p_2 - p_m\). This is due to that the bank will fail at the probability of \(1 - p_2 - p_m\), and the government has to pay $1 to the bank to avoid the systemic cost. Note that the market impact cost may or may not be proportional to the bank’s asset size. A financial institution with a smaller asset size but very convoluted relationships with other firms and banks may have a significant systemic cost when it fails. For convenience, we sometime refer to these banks with a higher market impact cost as “big” banks.

If these are the only two options, then as long as \((1 - p_1 - p_m)C(1 - p_2 - p_m)\) it is cheaper for the government to bail out a failed bank.
**Proposition 1:** Assume that the government can only choose among two options: either to give a full subsidy or none. The government will have a two-tiered bailout policy: It promises to give a subsidy to those failed banks which have a market impact larger than \((1 - p_2 - p_m)/ (1 - p_1 - p_m)\), and no subsidy to those with a market impact smaller than or equal to \((1 - p_2 - p_m)/ (1 - p_1 - p_m)\).

The critical value of the market impact cost, \((1 - p_2 - p_m)/ (1 - p_1 - p_m)\), depends on the relative riskiness of the “bad” project vs. the “good” project. The higher the ratio is, the smaller the number of banks which will be considered “too big to fail.”

What happens when the government has an option to adopt a randomized strategy for the bailout? This kind of bailout is often interpreted as a policy with “constructive ambiguity.”\(^9\) Assume the bailout probability is at \(k\), if the bank’s project gets a bad outcome. Thus, the expected payoffs for the bank if it takes project 1 or 2 are, respectively:

\[
E(X_1) = p_1R_1 + (1 - p_1 - p_m)[(1 - k)(-1)]
\]

\[
E(X_2) = p_2R_2 + (1 - p_2 - p_m)[(1 - k)(-1)]
\]

In this case, a simple calculation shows that as long as

\[
0 \leq k \leq 1 - \frac{(p_2 - R_2) - p_1R_1}{p_1 - p_2} = \bar{k}
\]

the bank will choose the good project. Consequently, the total expected cost to the government is

\[
(1 - p_1 - p_m)k + (1 - p_1 - p_m)(1 - k)C = (1 - p_1 - p_m)(C + k(1 - C)).
\]

To minimize the total expected cost, the optimal bailout probability \(k\) is set at the upper bound \(\bar{k}\) as long as \(C \geq 1\). For \(C < 1\), if a randomized strategy is adopted, then \(k\) should be zero, which, of course, corresponds to no bailout from the government.

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\(^9\) See Freixas (1999) for a similar interpretation. However, we will later extend this interpretation by including the uncertainty about the size of the subsidy as well.
If a full subsidy (bailout) is offered, then the bank will always choose project 2. The total expected cost to the government will be $1 - p_2 - p_m$, as under this scenario the bank will choose to invest in the riskier project 2. When it fails with a probability of $1 - p_2 - p_m$, the government will pay $1 to bail out the bank.

Comparing a randomized bailout policy with a full bailout policy when the bank always chooses project 2, the government prefers the randomized bailout to the full bailout policy if and only if

$$
(1 - p_1 - p_m) \bar{k} + (1 - p_1 - p_m) (1 - \bar{k}) C \leq (1 - p_2 - p_m).
$$

In other words, if and only if

$$
C > \frac{(1 - p_2 - p_m) - (1 - p_1 - p_m) \bar{k}}{(1 - p_1 - p_m) (1 - \bar{k})},
$$

then the bank is considered “too big to fail”.

Comparing the randomized strategy with the no-bailout policy, the government regulator prefers a randomized strategy if and only if

$$
(1 - p_1 - p_m) \bar{k} + (1 - p_1 - p_m) (1 - \bar{k}) C < (1 - p_1 - p_m) C.
$$

That is $C > 1$.

Combining the results in (7) and (8), we have a “three-tiered” pecking-order theory of government bailout policy.

**Proposition 2:** Given the government regulator can choose no bailout, full bailout, and randomized bailout policies, there exists a pecking-order:
(1) for the banks which would have a large market impact, 
\[ C > C^* = 1 + \frac{(p_1 - p_2)^2}{(1-p_1-p_m)(p_2 R_2 - p_1 R_1)} \], the government will bail them out with 100% probability;

(2) for the banks with a medium market impact, \( 1 < C \leq C^* \), the government will adopt a randomized bailout policy;

(3) for the banks with an insignificant market impact, \( C \leq 1 \), the government will not bail them out.

**Proof:**

From Condition (7), we know that when 
\[ C > \frac{(1-p_2-p_m)-(1-p_1-p_m)\bar{k}}{(1-p_1-p_m)(1-\bar{k})} = C^* \], the bank will be bailed out by the government when it fails. From (5),

\[ 1-\bar{k} = \frac{p_2 R_2 - p_1 R_1}{p_1 - p_2} \].

Substituting this back to the expression of \( C^* \) completes the proof of part (1). The other statements are clear from the previous discussion. \[ \text{QED.} \]

Proposition 2 is consistent with the Federal Deposit Insurance Corporation Improvement Act (FDICIA) of 1991’s “constructive ambiguity” policy with the exception of “too big to fail.” The systemic impact cost of a bank’s failure, measuring the complexity and linkage of a bank’s relation with other banks and firms, is determined by the nature and operations of a bank. The critical value for determining whether the bank is “big” or not, \( C^* \), from the government’s point of view, depends on the risk of the safer project, \( (1-p_1-p_m) \), and the spread of the up-side potential of the riskier project 2 and
the safer project 1, \( p_2 R_2 - p_1 R_1 \), and the risk spread between project 1 and project 2, \( p_1 - p_2 \).

Assume that the policy is consistent and that banks will not take the opportunistic approach, which will be discussed later, then it is quite obvious that an element of “constructive ambiguity” is desirable.

**Proposition 3:** Overall, the introduction of a randomized bailout strategy is welfare enhancing:

(1) more banks will choose good, safer projects; (2) the overall total expected costs for the government will be reduced.

**Proof:** We have derived in Proposition 2 that when \( C \leq C^* \), banks will not pursue risky projects under the randomized bailout strategy. Without the randomized strategy, only banks with \( C \leq (1 - p_2 - p_m) / (1 - p_1 - p_m) \) will not take risky projects. Thus, the first part is valid if \( C^* > (1 - p_2 - p_m) / (1 - p_1 - p_m) \). To show this, we need to prove that

\[
1 + \frac{(p_1 - p_2)^2}{(1 - p_1 - p_m)(p_2 R_2 - p_1 R_1)} > \frac{1 - p_2 - p_m}{1 - p_1 - p_m},
\]

which is equivalent to

\[
\frac{(p_1 - p_2)^2}{(1 - p_1 - p_m)(p_2 R_2 - p_1 R_1)} > \frac{p_1 - p_2}{1 - p_1 - p_m}.
\]

But this inequality is satisfied because \( p_2 R_2 - p_1 R_1 < p_1 - p_2 \) from (4).
For the second half of the proposition, assume a uniform distribution of $C$ on $[0, \overline{C}]$. Integration of the costs yields the result. With a randomized strategy described in Proposition 2, the total expected cost to the government is:

$$\frac{1}{C} \left[ \int_0^C (1 - p_1 - p_m) C dC + \int_{C^*}^C (\overline{k} + (1 - \overline{k})C) dC + \int_{C^*}^C (1 - p_2 - p_m) dC \right]. \quad (9)$$

With a full guaranteed subsidy strategy for banks with $C > \frac{1 - p_2 - p_m}{1 - p_1 - p_m}$, as described in Proposition 1, the total expected cost to the government is:

$$\frac{1}{C} \left[ \int_0^{1 - p_2 - p_m} (1 - p_1 - p_m) C dC + \int_{1 - p_2 - p_m}^{C^*} (1 - p_2 - p_m) dC \right]. \quad (10)$$

The difference in the expected costs to the government is $(10)$ minus $(9)$, which is the marginal benefit of introducing a randomized element in the overall bailout policy.

$$\text{Cost Diff.} = \frac{1}{C} \left[ \int_0^{1 - p_2 - p_m} \left( (1 - p_1 - p_m) C - \left( \overline{k} + (1 - \overline{k})C \right) \right) dC + \int_{1 - p_2 - p_m}^{C^*} \left[ (1 - p_2 - p_m) - \left( \overline{k} + (1 - \overline{k})C \right) \right] dC \right] > 0.$$ QED.

The reduction of the expected costs to the government by introducing a randomized strategy in the bailout can be seen from Figure 2. The expected cost savings to the government due to the introduction of the randomized strategy is the shaded area.
4: Role of Monitoring and Partial Rescue

In the above analysis, we have not considered the role of monitoring. The purpose of the monitoring and penalty scheme is to ensure that the bank will have an incentive to choose the right project to begin with and reduce the overall expected costs to the government.

In this section we will first assume that the verification of project choice occurs only after the bad outcome is realized. Therefore, we are looking for the second best outcome. Although monitoring and penalty in the up-state may get to first best, that is not what is happening in practice. A review of recent cases, ranging from Barings to Société Générale to National Australian Bank's FX trading losses to recent US bank failures all seems to point out that the governments have had "hands off" approach when things were "going well" and then ex post regret when there was a downturn where it was discovered that the banks' portfolios were loaded with risk. Also, in the bank regulatory
practice, off-site, ongoing monitoring (using quarterly call report) is not very accurate. If a bank engages in highly risky investment projects and lies about it, often the regulators will not know what is going on until the bad outcome is realized. Therefore, this is perhaps a realistic way to model what happens in the banking practice. At the end of this section, we will also describe what may be optimal if the government monitors the upside states (good and medium) to achieve the first-best solution.

4.1 Verification only After Bank Failure

In the current second-best monitoring and penalty regime, the verification will happen only when the bad outcome is realized. If a bank fails as a consequence of choosing project 1 (“good” one), then there is no penalty. If the bank is found to be guilty of taking project 2 (“bad” one), then a penalty, \( \Delta \), has to be levied as a deterrent for this excessive risk-taking behavior. The form of penalty can be pecuniary or non-pecuniary. For example, if found to be guilty of taking excessive risk, the bank and/or its managers may be fined, the managers may be barred from further directorship of other financial institutions, or face civil/criminal investigations. Alternatively, if the penalty is monetary, then the penalty can be interpreted as a “partial” rescue. In this case, if the outcome is bad, i.e., the net payoff is \(-1\), then the government will give the bank \(1 - \Delta\), provided doing so will still make the bank solvent. As we are discussing the penalty under the framework of a one-shot game, the penalty \( \Delta \) may be considered a one-shot penalty as well. In the next section, when we consider the dynamic implications of the bailout policy, we need to introduce the long-term penalty in a different context.

Assumption 1:

Ex ante, the government cannot force a bank to take a particular project; but
Ex post, when the bad outcome is realized, the government can verify which project is taken and give out a subsidy or a penalty accordingly.

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10 The system and its accuracy of the government regulators’ monitoring have been discussed in Cole and Gunther (1998) and Sierra and Yeager (2004). Demirguc-Kunt and Detragiache (2000) propose a duration based model to monitor banking risk.
Since our focus is the bank’s choice of project, the on-going monitoring for capital requirements and other operations are abstracted away from the model, although in practice, it is needed. In the current framework, it does not make any difference between monitoring a bank’s choice before or at the date \((t = 2)\) when the outcome of the project is realized\(^{11}\). Therefore, the term of monitoring is effectively \textit{ex post} auditing (verification).

We first consider a penalty scheme attached to the full bailout policy. We call it the transparent policy. If such a penalty is sufficiently large, then the explicit subsidy & penalty could effectively deter the bank from choosing the bad project. To see this, the revised net payoff for the bank (or its executives) is:

\[
X_1 = \begin{cases} R_1, & \text{prob.} = p_1 \\ 0, & \text{prob.} = p_m \\ 0, & \text{prob.} = 1 - p_1 - p_m \end{cases}
\]

and,

\[
X_2 = \begin{cases} R_2, & \text{prob.} = p_2 \\ 0, & \text{prob.} = p_m \\ -\Delta, & \text{prob.} = 1 - p_2 - p_m \end{cases}
\]

respectively.

In this case, as long as the penalty is

\[
\Delta \geq \frac{p_2 R_2 - p_1 R_1}{1 - p_2 - p_m},
\]

then the bank will have no incentive to take the “bad” project, irrespective of the bank’s systemic cost \(C\). While any penalty which is greater than \(\frac{p_2 R_2 - p_1 R_1}{1 - p_2 - p_m}\) will work effectively, there is no need for the government to impose an unnecessarily high amount, because this would lower the expected profits for the investors without any additional benefits. Therefore, we choose the lowest effective penalty \(\Delta = \frac{p_2 R_2 - p_1 R_1}{1 - p_2 - p_m}\) as the

\(^{11}\) It is possible that the error may depend on when the monitoring is conducted. For example, perhaps the monitoring is less accurate if it is conducted before \(t=2\) than otherwise. The extension in this direction may be conducted later.
optimal amount. Note that, from (4), \( \Delta = \frac{p_2 R_2 - p_1 R_1}{1 - p_2 - p_m} < \frac{p_1 - p_2}{1 - p_2 - p_m} < 1 \), which explains the reason why a penalty scheme can also be interpreted as a “partial” rescue under certain circumstances.

To elaborate on this point, note that the purpose of the government rescue is to keep the bank as a going concern while eliminating the incentive to take on riskier projects. Thus, the effective penalty \( \Delta \) should meet the solvency test. If the solvency of the bank requires that the minimum payoff from the project is maintained at the level of \( 1 - \Delta_1 \) \(^{12} \), then the total penalty given to the bank should have two parts: the pecuniary penalty given to the bank is limited to the level of \( \Delta_1 \) and the non-pecuniary penalty given to the bank amounts to \( \Delta_2 \), such that \( \Delta = \Delta_1 + \Delta_2 \). If the managers represent bank investors’ interest, as we have maintained in this paper so far, then \( \Delta_2 \) could be the penalty (both pecuniary and non-pecuniary) levied against the managers personally.

Combining this discussion with the results in Proposition 3, which states that large banks have incentives to choose riskier projects, we conclude that it is necessary to include the monitoring and penalty scheme for large banks.

**Proposition 4:** With a perfect monitoring technology, the following bank bailout policy can achieve ex ante efficient solution:

1. For the banks which have a large market impact \((C > C^*)\), the government will bail them out with 100% probability with ex post monitoring. If the bank is found to take the risky project 2, then a penalty of \( \Delta \) will be levied and the government essentially provides only a partial rescue;

2. For the medium firms \((1 < C \leq C^*)\), the government will adopt a randomized bailout policy;

3. For the smallest firm \((C < 1)\), the government will not bail them out.

\(^{12}\) This could be interpreted as the cash-flow based requirement for solvency.
The first best policy solution proposed in Proposition 4 guarantees that all firms will choose the less risky, better projects, and the regulator’s expected expenditures are minimized. The government still has to bail out some banks (large and some medium sized banks) when the failures of these banks are caused by nature in choosing the good project\textsuperscript{13}.

Furthermore, for the medium firms, the rescue could be partial as well. It is feasible that the government subsidy will bring the payoff, $m$, to the bank at the level of anywhere between $1-\Delta$ and 1. Then the probability of bailout $k$ should be changed accordingly as well. Assume the bailout probability is at $k$, if the bank’s project gets a bad outcome. Thus, the expected net payoffs for the bank if it takes project 1 or 2 are, respectively:

$$E(X_1) = p_1R_1 + (1-p_1-p_m)[k(m-1) + (1-k)(-1)]$$
$$E(X_2) = p_2R_2 + (1-p_2-p_m)[k(m-1) + (1-k)(-1)]$$

A simple calculation shows that as long as

$$0 \leq k \leq \frac{1}{m} \left[ 1 - \frac{(p_2R_2 - p_1R_1)}{(p_1 - p_2)} \right]$$

the bank will choose the good project. This result shows that the government regulator could choose both the level of the subsidy and the probability of the bailout for the medium sized firms. This fits the description of the “constructive ambiguity” very well.

But what happens when there is a limit to how far the government can make such a clear-cut \textit{ex post} verification about whether the bank has selected a good or bad project? Now let us specify an alternative assumption that the government’s monitoring is not 100% accurate.

\textbf{Assumption 2:}

\textit{Ex ante}, the government cannot force the bank to take a particular project; and

\textit{Ex post}, the government cannot verify which project is taken with 100% accuracy.

The government has a probability of $q$ making a mistake in monitoring. That is,

\textsuperscript{13}Note that we have assumed that the monitoring cost is zero. The government will initially bear the cost of monitoring, when the \textit{ex post} auditing is conducted, since the monitoring is costly. However, it can be internalized.
for a probability of \( q \), the bank taking a good project will be penalized nevertheless (type 1 error); and symmetrically, for a probability of \( q \), the bank taking the bad project will not be penalized (type 2 error).\(^{14}\)

\( q = 0 \) correspond to perfect monitoring. Under the full subsidy policy with imperfect monitoring, taking the “good” project will give the net expected payoff of

\[
E(X_1) = p_1R_1 - (1-p_1-p_m)q\Delta
\]

(11)

Taking the “bad” project will yield the expected net payoff to the bank,

\[
E(X_2) = p_2R_2 - (1-p_2-p_m)(1-q)\Delta
\]

(12)

It is obvious that the penalty has to be sufficiently large to make the banks better off by choosing project 1 and it is related to the degree of imperfect monitoring, \( q \). That requires,

\[
[(1-p_2-p_m)(1-q)-(1-p_1-p_m)q]\Delta \geq p_2R_2 - p_1R_1.
\]

(13)

Provided that

\[
q < \frac{1-p_2-p_m}{(1-p_1-p_m)+(1-p_2-p_m)},
\]

(14)

the effective penalty should be:

\[
\Delta \geq \frac{p_2R_2-p_1R_1}{(1-p_2-p_m)-[(1-p_1-p_m)+(1-p_2-p_m)]q} = \Delta^*(q).
\]

(15)

If the monitoring error is not too large, i.e., Condition (14) is satisfied, then from (15), \( \Delta^*(q) \) is an increasing function of \( q \) with an increasing rate, since both the first and the second derivatives of \( \Delta^*(q) \) with respect to \( q \) are positive. For the government to effectively deter banks from choosing the “bad” project, the optimal penalty should be positively related to the parameter \( q \).

**Proposition 5:** When the monitoring error is not too large, i.e. Condition (14) is satisfied, there exists a positive, convex relationship between the amount of penalty \( \Delta^* \) and the monitoring error \( q \).

\(^{14}\) Note, this assumption of symmetry of error in monitoring is innocuous. It will not affect the main results if the probability of type 1 error is different from that of type 2 error.
An explicit, full subsidy policy for large banks requires perfect \textit{ex post} verifiability on the government’s behalf. In an increasingly uncertain world where financial innovations take place all the time, the monitoring by the government’s regulatory agency cannot be perfect. The government regulator is often one step behind the market. Recent turmoil in the financial market, in particular, the innovations in mortgage-backed securities, offers a good example. Therefore, Proposition 5 calls for an increased penalty in a volatile, opaque market environment.

When the pace for financial innovation is increasing, and the penalty policy is fixed and rigid, the full subsidy and penalty policy is increasingly ineffective. In a more opaque financial environment, a heavier penalty is required to deter opportunistic behavior. When $q$ is increasing and the threshold for penalty cannot be adjusted on time, banks may risk taking “bad” projects. For example, if the government’s penalty policy is designed with a forecasted monitoring error at 10%, while the bank, closer to the frontier of financial market innovations, believes the monitoring error is at 20% then even with the explicit penalty, there is still an incentive for the bank to take the riskier project.

\textbf{Proposition 6:} \textit{When there is information asymmetry about the regulator’s monitoring precision, banks may take riskier projects if they believe that the actual monitoring error is bigger than that implied by the penalty scheme.}

Proposition 6 suggests that the government regulator has a tendency to play a catch-up game with banks in such a penalty scheme. Even if the regulator wants to play it safe by adding a premium on top of the critical value of the penalty, it will not fundamentally change the statement. Note that, increasing the penalty unnecessarily high will increase the deadweight cost and reduce the overall shareholders’ wealth.

What happens if the monitoring is so extremely unreliable that Condition (14) is no longer valid? Then any positive amount of penalty will not be effective because the right-hand side of the inequality of (13) is always negative, while the left-hand side is positive. As a matter of fact, a penalty which is very unreliably monitored will encourage risk taking.
**Proposition 7:** If the monitoring error is sufficiently large, i.e. Condition (14) is violated, then the government should give up the penalty scheme.

### 4.2 Monitoring in the Solvent States

We are now exploring the issue of monitoring done by the government regulator when the bank is solvent. A suspiciously high net cash flow, as indicated by the cash flow $R_2$ in the model, indicates clearly that the bank takes a riskier project 2. However, as there are two possible solvent states, “medium” and “high”, an inspection on the cash flow of the bank during the solvent state may not be fully revealing when the state of nature is in the medium state (i.e. the net cash flow is zero). As both the safer project and the riskier one may achieve the same “medium” outcome, a further costly investigation is needed. The result of such an investigation may not be clear-cut. Therefore, the key issue here is the trade-off between the cost of monitoring and the incentives offered to the bank for taking less risky project 1, with the adjustment for monitoring errors. The underlying rationale follows the principle of costly state verification (CSV) developed by Townsend (1979).

Similar to the previous discussion when the verification is conducted only in the upper state, an *ex ante* bailout policy can be designed. We will assume that (1) the monitoring in the solvent state is more complicated and with a larger monitoring error; and (2) such monitoring is costly. Specifically, as both projects yield the same net payoff in the medium state, we assume that based on this cash flow information, the regulator may have much difficulty to determine which project the bank takes, therefore, the monitoring error $q$ is far larger than the monitoring error $q$ occurred when the bank fails.\(^\text{15}\) The effective penalty will be adjusted to $\tilde{\Lambda}$ accordingly.

Take a case when the bank takes the “bad” project and the monitoring takes place irrespective whether the bank is solvent or not. If the high cash flow $R_i$ is detected, then the firm is fined at the amount of $\tilde{\Lambda}$. If the medium state is realized, then the regulator

\(^\text{15}\) This assumption makes sense since if the bank is indeed insolvent, its operations will get more scrutiny from other creditors and news media, which helps the government investigation.
will commit a monitoring error of $\tilde{q}$, which is both the scale of the type-1 error and type-2 error. If the bad outcome is realized, then the regulator will incur a monitoring error of $q$, $q < \tilde{q}$. Therefore, the net payoff to the bank is (1) in the “high” state, $R_2 - \tilde{\Delta}$ when $X_2 = R_2$; (2) in the “medium” state, either 0 (with probability of $p_m \tilde{q}$, because the regulator fails to detect the risky project taken) or $-\tilde{\Delta}$ (with probability $p_m (1 - \tilde{q})$); (3) in the “low” state, either 0 (with probability of $(1 - p_2 - p_m)q$, because the regulator fails to detect the risky project taken) or $-\Delta$ (with probability $(1 - p_2 - p_m)(1 - q)$). Thus,

$$X_2 = \begin{cases} R_2 - \tilde{\Delta} & \text{prob. } = p_2 \\ 0 & \text{prob. } = p_m \tilde{q} + (1 - p_2 - p_m)q \\ -\tilde{\Delta} & \text{prob. } = p_m (1 - \tilde{q}) + (1 - p_2 - p_m)(1 - q) \end{cases}$$

A complete list of the bank’s net payoff under two different monitoring regimes is in Table 1.

[Please Insert Table 1 Here.]

It is relatively straightforward to derive the ex ante optimal bailout policy as in Section 4.1, with a large bank will incur a penalty of $\tilde{\Delta}$ if it is found to be taking the risky project 2. Denote the required minimum effective penalty $\tilde{\Delta}^*$ $(q, \tilde{q})$. Proposition 8 compares it with $\Delta(q)$ derived in Equation (15).

**Proposition 8:** Assume $\tilde{q} < \max \left( \frac{p_m + p_2}{2p_m}, 1 \right)$, the minimum required effective penalty under the all-state monitoring regime is smaller than that required under the bad-outcome monitoring regime. That is, $\tilde{\Delta}^*(q, \tilde{q}) < \Delta(q)$.

**Proof:** To deter the bank from taking project 2, it is required that,
\[ p_1 R_1 - \tilde{\Delta} (p_m \tilde{q} + (1 - p_1 - p_m)q) \geq p_2 (R_2 - \tilde{\Delta}) - \tilde{\Delta} (p_m (1 - \tilde{q}) + (1 - p_2 - p_m)q). \]

Thus,
\[ \tilde{\Delta} \geq \frac{p_2 R_2 - p_1 R_1}{(1 - p_2 - p_m) - [(1 - p_1 - p_m)\big((1 - p_2 - p_m) + (1 - p_2 - p_m)\big)]q + p_m(1 - 2\tilde{q}) + p_2} = \Delta^* (q, \tilde{q}) \quad (16) \]

Compare the denominator in (16) with that in (15), we obtain the desired result.

Q.E.D.

We notice that both monitoring regimes could be designed such that ex ante, banks have no incentive to take a risky project. As the monitoring costs are higher and the monitoring errors are larger when the monitoring is done in all states of nature, the government regulator has to decide whether to do so based on the trade-off of the additional monitoring costs and the reduced bailout costs.

5: Credibility of the ex ante Bailout Policy

We now turn to the dynamic implication of the optimal bailout policy, derived in the previous sections. The optimal bailout policy as illustrated in Proposition 4 suffers a credibility problem. Banks which are not “too big to fail” may act as if they belong to this category. When this happens, will the government bails them out nevertheless? We will argue that both the bank and the regulator contribute to this problem.

There are two cases where the bank may take an opportunistic approach: one is when the bank is in the last period of its life, which is not a particularly interesting scenario. The other is when the bank perceives that the government cares too much about the short-term cost and may abandon its commitment for the pre-specified bailout policy. For example, when a medium sized bank perceives that the probability of being bailed is higher than \( k \) and switches to project 2 accordingly, then the ex ante probability of failure is increased from \((1 - p_1 - p_m)\) to \((1 - p_2 - p_m)\). When the bad state is realized ex post, the cost of the bailout is $1, but to refuse to provide the bailout will have the systemic impact of \( C \). Therefore, all the banks with \( C > 1 \), are willing to bet that the short-term oriented government will rescue them ex post.
If the government has its long-term interest in mind, it will stick to its commitment in the bailout policy. However, a government worrying about the short-term cost may renege on its commitment. The government focus on short-term vs. long-term could be explained by many factors such as, whether the government budget is in surplus or in deficit; overall macro-economic conditions; the election cycle; and the difference between an authoritarian government and a democratically elected government, among others. Although these factors are important in public policy, a complete analysis of them is beyond the scope of modeling in this paper. Instead, the following investigation assumes that the government may not be able to fulfill its commitment to a pre-specified optimal bailout policy, but has good memory and may implement a long-lasting penalty, such as black-listing the bank by cancelling any preferential treatment or contracts in the future. Then the dynamic consistency problem could be analyzed following Friedman’s (1971) “trigger strategy” and its variants.

Assume that a bank with a medium or large impact cost \((C > 1)\) has an infinite life. In each period, it faces the project choice described in previous sections. At a particular period \(s\), it considers whether to act opportunistically and take the risky project anyway. If it takes the “riskier” project and the government bails it out, then the one-shot expected gain is \(p_2 R_2 \left( p_1 R_1 - (1 - p_1 - p_m) (1 - \Delta) \right)\).

The government needs to do ex post monitoring (auditing) for the failed medium-sized institutions. If the medium-sized firms are found to be guilty of taking riskier projects, they will be bailed out this once, because a government with a short-term orientation is intent on reducing the cost of the impact on the market. However, those banks will suffer long-term consequences. They will lose favor with the government forever, and lose an amount equivalent to \(\Pi\) for every period after \(s\). Assume that the bank has a discount rate of \(\delta\). Then it will take the riskier project at \(s\) if and only if:

\[
p_2 R_2 \left( p_1 R_1 - (1 - p_1 - p_m) (1 - \Delta) \right) > \frac{\Pi}{\delta},
\]

The right-hand side of (16) is the one shot gain for the bank, while the left-hand side is the present value of a perpetuity of the (per period) penalty, \(\Pi\), discounted at the rate of \(\delta\).
**Proposition 9:** Both the ambiguous and full bailout policies have credibility problems for any firm with a $C > \$1$. Consequently, for a smaller bank ($C < \$1$), there is no need for government monitoring, because there is no incentive for it to take risky projects. A larger bank will always take the opportunistic approach if it is in the end period. If a larger bank’s life is infinite, then such opportunistic behavior could be constrained by a reputation factor, which requires an effectively long-lasting penalty from the government for any violation.

6: Systemic Impact Cost as a Choice Variable

So far, we have assumed that the bank is endowed with an exogenously determined systemic cost $C$. However, in reality, $C$ may be endogenously determined as well. Acharya (2001) focuses on the bank’s endogenous choice of its systemic cost through the correlation of its returns with the returns of other banks. Our model offers a different explanation in the endogenization of the market impact cost: smaller banks may merge with each other or expand to other lines of business to increase their impact cost and qualify for government subsidies, in line with the three-tiered “pecking-order” theory of the government bailout. This gives a theoretical foundation to the empirical findings by Penas and Unal (2004), which find that reduced costs in the bond market are an incentive for mid-size banks to merge and thus become “too big to fail”. Earlier evidences can be found in O’Hara and Shaw (1990), Hughes and Mester (1993) and Angbozo and Saunders (1996). Other firms in the manufacturing sector may expand their employment to suboptimal level and hope to avoid the financial distress by threatening to close factories and lay off people.

Analytically, a bank will trade off the benefit of the increased likelihood of a government subsidy and the costs of engaging in a merger or expansion. The optimal size of the firm is therefore partially determined by the government bailout policy. In this sense, we conclude that the boundary of the firm can be expanded by the government regulatory policy.

The existence of a government subsidy scheme may also affect a firm’s risk management practice and risk sharing among firms. First, there is a reduced incentive for
the firms to manage risk if doing so will reduce the market impact cost and therefore the expected subsidy from the government. Second, the type of the risk sharing among the banks will also be reduced. For example, a bank choosing project 2 may be less correlated with a similar type of project of another bank. Both banks could share the risk without the involvement of the government. With the existing arrangement of the government bailout policy, such incentives for banks to share the risk are lost in the process.

7: Conclusions

In this paper, we have derived the ex ante optimal bailout policy when a bank’s failure has systemic impact. The optimal policy calls for a guaranteed bailout with a penalty attached for banks whose failure would have the greatest market impact, a randomized bailout for banks with medium impact and no bailout for banks with least market impact. Large banks with guaranteed bailout are subject to government ex post monitoring to check whether they are prudent in project (loan portfolio) choice. As financial innovations increase, the opacity and difficulty of monitoring may provide banks with the incentive to take excessive risks if the penalty scheme is fixed. Banks with medium market impact may take an opportunistic approach because they expect a short-term oriented government to bail them out ex post. To deter this opportunistic behavior, a long-term penalty is required. The bailout strategies which would give nothing to smaller banks should increase the pressure for them to merge with each other or expand into different lines of business to attract government bailout subsidies.
Appendix

1: Proof that Project 2 is riskier than project 1.

For $i = 1, 2$,

$$E(X_1) = p_i R_i - (1 - p_i - p_m), \text{ and } \sigma_i^2 = p_i (1 - p_i) (1 + R_i)^2 + p_m [1 - p_m - 2 p_i (1 + R_i)].$$

$$\sigma_2^2 - \sigma_1^2 = p_2 (1 - p_2) (1 + R_2)^2 - p_1 (1 - p_1) (1 + R_1)^2 + 2 p_m [p_1 (1 + R_1) - p_2 (1 + R_2)].$$

Since

$$p_2 (1 - p_2) (1 + R_2)^2 - p_1 (1 - p_1) (1 + R_1)^2$$

$$= p_2 (1 - p_2) - p_1 (1 - p_1) + p_2 (1 - p_2) (2 R_2 + R_2^2) - p_1 (1 - p_1) (2 R_1 + R_1^2)$$

$$= (p_1 - p_2)(p_1 + p_2 - 1) + p_2 (1 - p_2) (2 R_2 + R_2^2) - p_1 (1 - p_1) (2 R_1 + R_1^2)$$

$$> (p_2 R_2 - p_1 R_1)(p_1 + p_2 - 1) + p_2 (1 - p_2) (2 R_2 + R_2^2) - p_1 (1 - p_1) (2 R_1 + R_1^2)$$

$$= p_1 p_2 R_2 - p_1^2 R_1 + p_2^2 R_2 - p_1 p_2 R_1 - p_2 R_2 + p_1 R_1 + 2 p_2 R_2 - 2 p_2^2 R_2 + p_2 R_2^2 - p_2^2 R_2^2$$

$$- 2 p_1 R_1 + 2 p_1^2 R_1 - p_1 R_1^2 + p_2^2 R_2^2$$

$$= p_1 p_2 (R_2 - R_1) + p_2 R_2 (1 - p_2) - p_1 R_1 (1 - p_1) + p_2 R_2^2 (1 - p_2) - p_1 R_1^2 (1 - p_1)$$

$$> p_1 p_2 (R_2 - R_1) + (p_2 R_2 - p_1 R_1) (1 - p_1) + (p_2 R_2 - p_1 R_1) R_1 (1 - p_1) > 0.$$ and

$$[p_1 (1 + R_1) - p_2 (1 + R_2)] = (p_1 - p_2) - (p_2 R_2 - p_1 R_1) > 0,$$

Therefore, $\sigma_2^2 > \sigma_1^2$. \[QED\]
References:


Morgan, Donald P. and Kevin J. Stiroh, 2005: "Too big to fail after all these years," Staff Reports 220, Federal Reserve Bank of New York.


Table 1: A comparison of the net payoff to the bank: monitoring is conducted both in solvent states and insolvent states vs. monitoring is conducted only in the insolvent state

$\Delta$ is the penalty given to the bank for taking risky project 2 when the verification is only conducted when the bank fails; while $\tilde{\Delta}$ is the penalty given when the monitoring is conducted irrespective of the bank’s solvency. $q$ is the monitoring error when the verification is conducted in the low state of cash flows; while $\tilde{q}$ is the monitoring error when the verification is conducted when the cash flow reaches the medium state, respectively.

<table>
<thead>
<tr>
<th>Project</th>
<th>Net payoff (after penalty) to the bank when monitoring is only conducted in the insolvent state</th>
<th>Net payoff (after penalty) to the bank when monitoring is conducted in the solvent states as well</th>
</tr>
</thead>
<tbody>
<tr>
<td>“Good” Project:</td>
<td>$X_1 = \begin{cases} { R_1 \text{ prob.} = p_1, 0 \text{ prob.} = p_m, -1 \text{ prob.} = 1 - p_1 - p_m } \ \ X_1 = \begin{cases} R_1 \text{ prob.} = p_1, 0 \text{ prob.} = p_m + (1 - p_1 - p_m)(1 - q) - \Delta \text{ prob.} = (1 - p_1 - p_m)q \end{cases} \ X_1 = \begin{cases} R_1 \text{ prob.} = p_1, 0 \text{ prob.} = p_m(1 - \tilde{q}) + (1 - p_1 - p_m)(1 - q) - \tilde{\Delta} \text{ prob.} = p_m \tilde{q} + (1 - p_1 - p_m)q \end{cases} \end{cases}$</td>
<td>$X_1 = \begin{cases} R_1 \text{ prob.} = p_1, 0 \text{ prob.} = p_m + (1 - p_1 - p_m)q - \Delta \text{ prob.} = (1 - p_1 - p_m)(1 - q) \end{cases}$</td>
</tr>
<tr>
<td>“Bad” Project:</td>
<td>$X_2 = \begin{cases} { R_2 \text{ prob.} = p_2, 0 \text{ prob.} = p_m, -1 \text{ prob.} = 1 - p_2 - p_m } \ \ X_2 = \begin{cases} R_2 \text{ prob.} = p_2, 0 \text{ prob.} = p_m + (1 - p_2 - p_m)q - \Delta \text{ prob.} = (1 - p_2 - p_m)(1 - q) \end{cases} \end{cases}$</td>
<td>$X_2 = \begin{cases} R_2 - \tilde{\Delta} \text{ prob.} = p_2, 0 \text{ prob.} = p_m \tilde{q} + (1 - p_2 - p_m)q - \tilde{\Delta} \text{ prob.} = p_m(1 - \tilde{q}) + (1 - p_2 - p_m)(1 - q) \end{cases}$</td>
</tr>
</tbody>
</table>